The Condorcet paradox: an experimental approach to a voting process

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Abstract

This paper analyses the effects played by rules within a coordination game. The starting point is constituted by the wide field of Public Choice theories. More precisely the focus of the research is on the stability of the voting process. The experiment is build on a game played through computers and the experimental subjects must perform some choices that can led to different individual and collective solutions. The game that they play is based on a set of rules that must be voted by the players themselves before a new session of the experiment will be run. The idea is to verify the degree of stability of the collective choices (log-rolling phenomena).

1. Introduction

This paper is on the Condorcet paradox. More precisely is an experimental investigation on the stability of the voting process. The idea is to test the well known phenomenon of cyclical voting that should arise whenever the voters have double peaked preferences.

The Condorcet paradox is: "a situation described by the Marquis de Condorcet in the late 18th century, in which collective preferences can be cyclic (i.e. not transitive), even if the preferences of individual voters are not. This appears paradoxical, because it means that majority wishes can be in conflict with each other. This paradox can be explained away by the fact that in that case the majorities are made up of different groups of individuals. The paradox is highlighted by the Condorcet method of voting, which will fail to determine a winner in such a situation — an alternate technique must then be used"[1].

A simple way to describe the Condorcet paradox is to use a numerical example like the following (Rouncefield, Green, 2003):

Three players: A, B and C

Pr(A outscores B) = 5/9

Pr(B outscores C) = 5/9

Pr(A outscores C) = 4/9)

i.e. A is better than B i.e. B is better than C i.e. A in not better than C, which means that the results are not transitive.

An example of a real game that produces a Condorcet type situation is the so called Chinese Dice game (Green, 1981) The dice of this game are marked in this way:

Die A) 6, 6, 2, 2, 2, 2

Die B) 5, 5, 5, 5, 1, 1

Die C) 4, 4, 4, 3, 3, 3

For these:

Pr(A outscores B) = 5/9

Pr(B outscores C) = 6/9

Pr(A outscores C) = 3/9

Also this game produces results which are not transitive and therefore are cyclical.

The most important application of the Condorcet paradox is the study of the voting systems. The paradox arises every time the voters have a preferences structure which is individually transitive but collectively intransitive. For example imagine three voters that have the following preferences structures over three alternatives a_1, a_2, a_3 (e.g. three different quantities of some public good):

Voter	Preference
1	$a_1 \succ a_2 \succ a_3$
2	$a_2 \succ a_3 \succ a_1$
3	$a_3 \succ a_1 \succ a_2$

Assuming that the voters give their support to the first and second choice while never vote for the third choice, then a winner cannot exist for this profile if all the alternatives are individually voted.

In literature there are many examples of real situations where the Condorcet paradox takes place. I shall restrain myself to quote only Kurrlid-Klitgaard (2001) who describes the case of the existence of a real cyclical majority in a poll of Danish voters' preferred prime minister, using pair-wise comparison during the elections of the Danish prime minister in 1994. Kurrlid-Klitgaard (2001) is also useful for a short review of some articles based on empirical evidences of the Condorcet paradox.

Conversely looking to the experimental literature there are not many examples of experiments done on the voting process. The focus of the experimental researches is mainly concerned with the effects produced on the collective choices by different voting systems, instead than on the investigation of the role plaid by different preferences structures. Just as an example of an experimental study on the role played by different voting rules one could see Forsythe (1991) that analyses a three ways elections model.

The main attention is here concentrated on the central assumption made by the Condorcet paradox, i.e. on the effects produced by double peaked preferences on the stability of the voting process. This is a topic that is very difficult to investigate using a "traditional" empirical approach because the only practical way to check the preferences structure is to ask to the voters to declare spontaneously their wishes towards a given topic. Obviously the voters can have strategic reasons to make false declarations and in any way the real behaviours are not observable so there is no way to check the truthfulness of the individual declarations.

A way to overcome these limits is to use the experimental approach, which allows to simulate in an artificial environment a voting process. To simulate a voting process able to investigate on the effects produced by the Condorcet paradox requires three main ingredients:

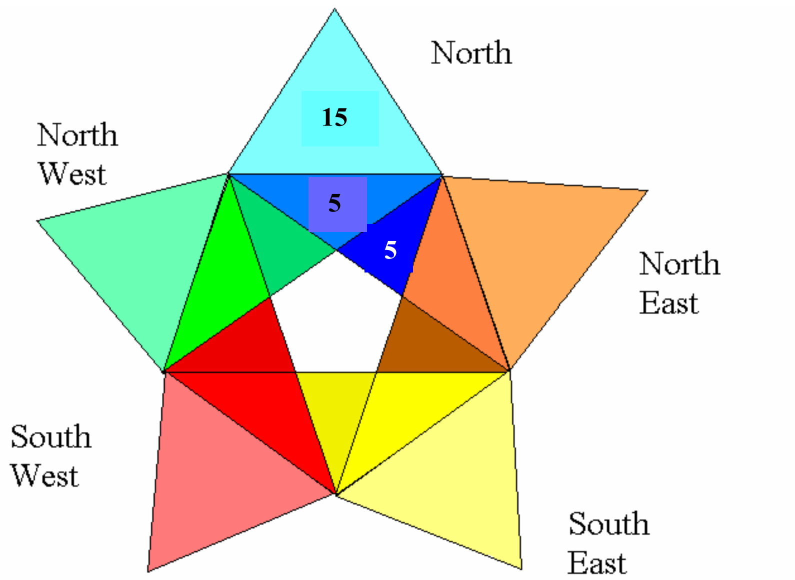
[1] http://en2.wikipedia.org/wiki/Condorcet's_paradox

a payoffs structure that links the utility of the experimental subjects to the voting choices, an artificial structure of preferences coherent with the payoffs-utility configuration and a social environment. The experiment here discussed owns all these characteristics.

2. The experimental design

The experiment is a repeated game played by five players. Each round is divided in two phases: the voting stage and the playing stage. In the voting stage the experimental subjects must choose (and vote) a rule that will be used in the game stage. To play the game the subjects must rotate one among four geometric figures that shape a pentagon star. In fig. 2.1 is reported the form used for the game.

Fig. 2.1 The pentagon star



The first geometrical figure is the large pentagon star which includes the five external triangles, the second figure is the inscribed pentagon which is made by the five intermediate triangles, finally the third figure is the small star made by the five inner triangles. Each figure rotates anti-clockwise, independently from the other ones, and with single steps. The single step rule means that to move one triangle from its initial position back to the starting setting requires five moves (steps). For example to take the blue external triangle from the North position to the South West one needs three moves.

Each of the five players occupies a position indicated by a cardinal point. Subject 1 is in position North, subject 2 is in position North East, subject 3 is in position South East, subject 4 is in position South West and finally subject 5 is in position North West. To each player corresponds a colored geometrical surface which is the sum of three triangles: the blue area for subject 1, the orange area for subject 2, the yellow area for subject 3 (yellow), the red area for subject 4 and the green area for subject 5.

To the colored surfaces correspond the payoffs areas. The players gain a final prize which is the sum of the values reported on each triangle. Only three triangles have numbers on them, therefore to win something one must rotate the figure which takes one of the numbered triangles in her/his payoff area. For example if it is the turn of player North West the most rational move is to rotate the external star which will attribute to her/him a payoff of 15 points.

The general ingredients of the experiment are the following:

- five players, four groups of players;
- anonymity;
- a set of rounds – each round is divided into two phases: phase1 voting the rules; phase2 move the wheels accordingly with the rule voted by the majority; one rotation per player;
- the positions of the players are assigned randomly;
- after 3 voting sessions without a majority the experiment stops and the subjects win a fixed (small) reward.

The number of players can be obviously changed but a total of 25 subjects for each experimental session build up a reasonable sample. Similarly the anonymity condition is not a strict one and can be relaxed. In the experiments here discussed has been maintained to have a "cleaner" experimental outcome. When the subjects have no way to identify their partners the results are less affected by psychological uncontrolled factors like for example some form of sympathy or antagonism between two or more participants.

The rules that must be voted in the first phase of the experiment are the following:

(T) - All the wheels can be rotated[2]

(I) - only the Internal wheel can be rotated

[2] "T" stands for the Italian "Tutte", which means all.

(E) - only the External wheel can be rotated

The rules used for the experiment are intended to build a system of artificial preferences that models a Condorcet paradox situation. More precisely the artificial preferences needed are the following:

- Player North (N) $I \succ T \succ E$
- Players North and South East (NE; SE) $E \succ I \succ T$
- Players North West and South West (NW; SW) $T \succ E \succ I$

To obtain the desired artificial preference system the rules must be integrated with a special condition for player North. More precisely to be fair and to be coherent with the artificial preferences system the payoffs scheme must take this structure:

- each point is converted in Euro Cents
- 1 cent for player North
- 9 cents for players North East and South East
- 3 cents for Players North West and South West
- special payoff for player North: if the rule is T s/he will win 22 points if the light blue triangle goes to occupy the position South West.

The artificial preference structure just described holds only for the first move, i.e. in a one-shot game setting. To explore the dynamical solution of the game – assuming that each player moves only once for a total of 5 moves per round – is useful to write the numerical solution. Tab. 2.1 reports the solution of the game (assuming rational players), the second column of the table shows the moves that the players can do while the third column reports the individual payoff obtained by each player. Looking to tab. 2.1 is immediately clear that from a social point of view the most efficient rule is the "all wheels can rotate". At the same time it is also evident that the all wheels rule is the worst for both the North East and the South East players that never win when this rule is at work. The obvious consequence is that the East players should try to contrast the all wheel rule.

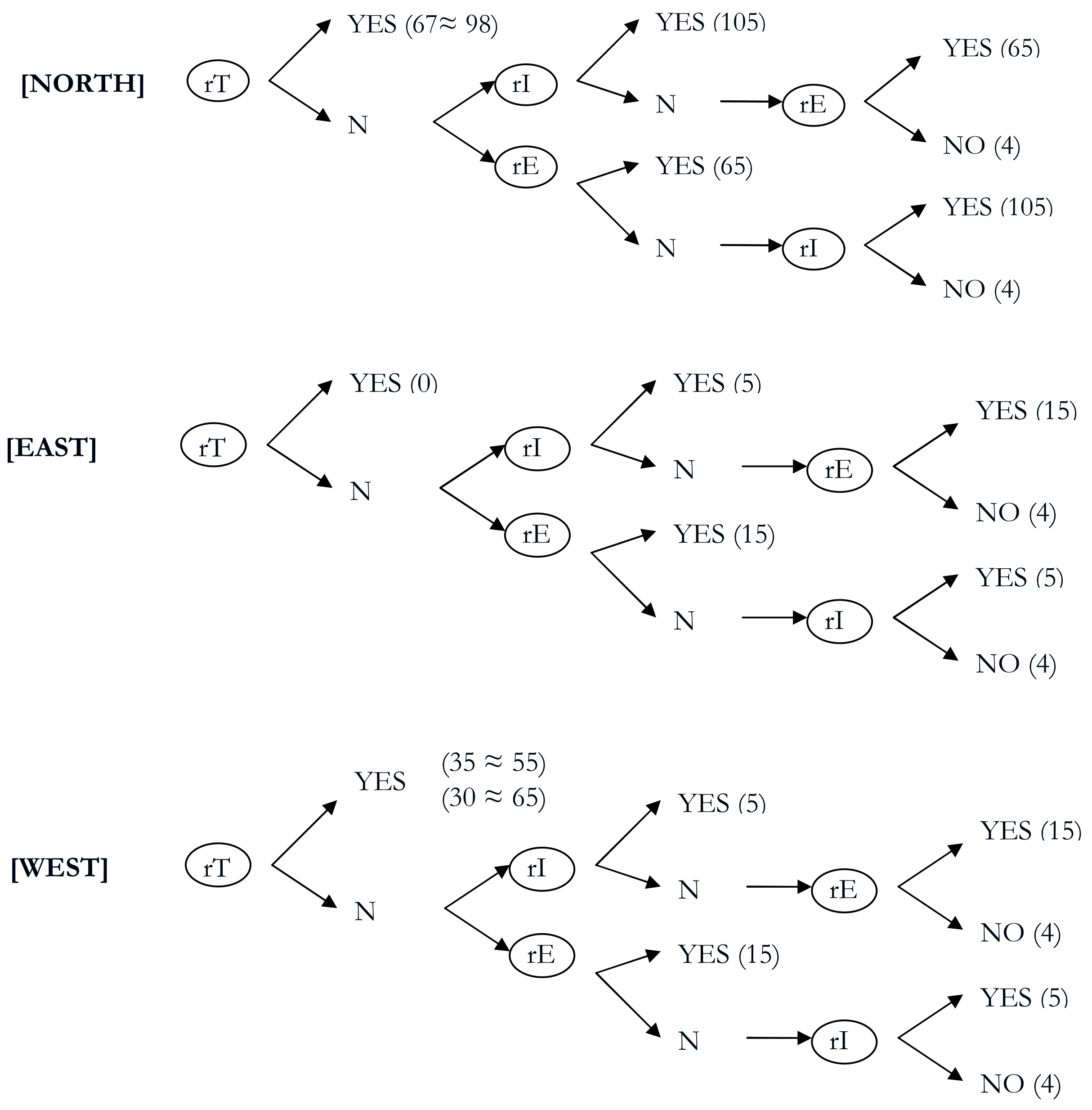
An even clearer demonstration of the dominance of the all wheels rule is given by the dynamic solution of the game. The solution of a session made of five moves (one for each player) is reported in fig. 2.2. It is important to underline that at the beginning of the experiment the players do not know the voting agenda, i.e. they do not know ex ante if they should vote first for rule all, then for rule external and then for the rule internal or for some different sequence of the rules. In particular, they do not know what rule will follow the first voting session.

Tav. 2.1 Rational moves											
Player	**Moves**					**Payoff**					**Pay Tot per round**
	NE	SE	SO	NO	N	N	NO	SO	NE	SE	
Rule All (T)	E	E	M	I	M	98	35	65	0	0	198
	E	E	M	I	I	98	35	65	0	0	198
	E	E	I	M	M	98	35	65	0	0	198
	E	E	I	M	I	98	35	65	0	0	198
	E	M	E	I	M	81	55	50	0	0	186
	E	M	E	I	I	81	55	50	0	0	186
	E	I	E	M	M	81	55	50	0	0	186
	E	I	E	M	I	81	55	50	0	0	186
	M	E	E	I	M	91	45	50	0	0	186
	M	E	E	I	I	91	45	50	0	0	186
	M	M	E	I	E	67	45	35	0	0	147
	M	M	I	E	E	77	35	35	0	0	147
	M	I	M	E	E	72	45	30	0	0	147
	M	I	I	E	E	72	45	30	0	0	147
	I	E	E	M	M	91	45	50	0	0	186
	I	E	E	M	I	91	45	50	0	0	186
	I	M	M	E	E	72	45	30	0	0	147
	I	M	I	E	E	72	45	30	0	0	147
	I	I	E	M	E	67	45	35	0	0	147
	I	I	M	E	E	77	35	35	0	0	147
Average											172,8
Rule External (E)	E	E	E	E	E	65	15	15	15	15	125
Rule Internal (I)	I	I	I	I	I	105	5	5	5	5	125

The final situation that emerges from the experimental design is not the one described by the Condorcet paradox because the cyclical nature of the voting process holds only in a one shot setting while in a dynamic strategic perspective the result should be of a steady state with the dominant rule always voted by the majority.

The experiment models a situation where the static preferences are double peaked and should take to a cyclical process of voting but the dominant dynamic strategy is stable so we expect that a rational output for the game is to lock in the “all wheels” rule.

Tab. 2.2 The dynamic solution



Sequences:

North NO_T YES_I YES_E	North 65
East NO_T NO_I YES_E	East 15
West NO_T NO_I YES_E	West 15

North NO_T NO_E	North 65
East NO_T YES_E	East 15
West NO_T YES_E	West 15

North YES_T $67 \approx 98$

East NO_T East 0

West YES_T West $35 \approx 55$; $30 \approx 65$

3. The results

Two experimental sessions have been run using two separate samples each of them made of 20 participants. The experimental subjects were students of the faculty of economics recruited through advertisements. Half of them were male and the other half female.

The difference between the two experimental sessions is exclusively related to the number of rounds, more precisely in the first experiment the participants had to approve one rule majority nine times, while in the second one the number of the voting sessions that must be closed with a majority on a rule was eighteen. It is worth remembering that the experiment stops when the participants did not reach a majority on a rule after 3 voting sessions, therefore 9 and 18 are the "potential" number of voting sessions. In spite of the constraint on the number of null voting sessions the experiment didn't really stop because we were interested in collecting a complete data set. Of course, the subjects perceived only a reduced payoff, accordingly with the general regulations of the experiment.

The subjects interacted through a computer screen, could not communicate and each of them was separated from the others with a box. Before the beginning of the experiment each participant received the instructions sheet which was also read by a researcher to be sure that everybody understood it correctly. Questions on the instructions were asked and answered publicly.

The results from the voting session are reported in table 3.1 while those from the second experiment can be found in table 3.2.. Both tab. 3.1 and tab.3.2 show four tables: one for each of the 4 groups made by 5 subjects. In the first column of each table are reported the players, N identifies the North player, NE the North East player, SE the South East player, SO the South West player and NO the North West player. The first row of each table shows the rules to be voted, it is worth remembering that T indicates the "all wheel rule", I is the "internal wheel rule" and E is "the external wheel rule". In the following columns one can see the results of the vote: zero means that the rule has not been voted by a given player, while one means that the player voted for the rule. The results shown by table 3.1 give a first piece of information on the voting decisions assumed by the participants of the first experiments. In all the groups a cyclical path takes place but it is almost always reduced to two alternatives: the rules that win the competitions are “the external wheel rule “ and the “all wheels rule”. This means that in spite of theoretical expectances, cyclical patterns emerge from the games. More precisely, the all wheels rule -which was expected to dominate the competition- has been often voted but the external wheel rule has also been voted for almost the same number of time (19 times the subjects voted the T rule versus 16 times the E rule). It is worth noticing that the internal wheel rule had the same cumulative performance, measured in terms of total payoff, than the external rule. The reason why the internal rule has been voted only once (group 1, round 2) is because it produced a strong advantage only for one player, i.e. player North.

Tab. 3.1 Results from the first experiment

Group 1

	Rules to be voted												
Players	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**
N	0	1	0	0	1	1	0	1	1	0	1	1	0
NE	0	0	0	1	0	0	0	0	1	1	0	1	0
SE	1	0	0	1	0	0	1	0	0	1	0	0	1
SO	1	1	1	1	0	1	1	0	1	1	1	1	1
NO	1	1	1	1	0	1	1	0	0	1	0	0	1
accepted	T	I	-	T	-	E	T	-	E	T	-	E	T

Group 2

	Rules to be voted												
Players	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**
N	1	1	1	1	1	1	1	1	1	1	1	1	1
NE	1	0	0	0	0	1	0	0	1	0	0	1	0
SE	1	0	1	0	0	1	0	0	1	0	0	1	0
SO	0	1	1	1	0	0	1	0	0	1	0	0	1
NO	1	0	1	1	0	0	1	0	0	1	0	0	1
accepted	T	-	E	T	-	E	T	-	E	T	-	E	T

Group 3

	Rules to be voted														
Players	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**
N	1	1	1	1	0	1	1	0	1	1	0	0	1	0	1
NE	0	1	0	0	1	1	0	0	1	0	0	1	0	0	1
SE	1	0	1	1	0	1	1	0	1	0	0	1	1	0	1
SO	1	0	0	1	0	0	1	1	0	1	1	0	1	0	0
NO	0	0	1	1	0	0	1	0	0	1	0	0	1	0	0
accepted	T	-	E	T	-	E	T	-	E	T	-	-	T	-	E

Group 4

	Rules to be voted														
Players	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**
N	1	0	1	0	0	1	1	1	1	1	1	1	1	1	1
NE	1	1	1	0	0	1	0	0	1	0	0	1	0	0	1
SE	1	0	1	0	0	1	0	0	1	0	0	1	0	0	1
SO	1	0	0	1	0	0	1	0	0	1	1	0	1	0	0
NO	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
accepted	T	-	E	-	-	E	T	-	E	T	-	E	T	-	E

A partially different result has been obtained from experiment 2 where group 2, 3 and 4 show a tendency towards a steady state in the voting mechanism. The first group of subjects of the second experiment shows not only a cyclical pattern of voting but also an almost complete cycle with the rule I voted by the majority of the participants 4 times out of 18 voting sessions. As just anticipated this is not the case for groups 2, 3 and 4. Group 2 locked in rule T, as the theory had forecasted, and the subjects belonging to this group voted a different rule only twice. A similar lock in process can also be observed for the first 10 voting sessions of group 3, which approved the T rule eight times voting the E rule only twice. Interestingly, after round 10, the players almost totally left the T rule starting to vote initially the I rule (twice) then locking in the E rule untill the end of the experiment. The group 4 pattern can be considered as the almost perfect mirror like image of group 3. The members of group 4 locked in the E rule for the first half of the experiments (voting the T rule only twice during the first 8 voting sessions) to pass to a steady state in favor of the T rule after round 9.

It is worth underlining that both group 2 (after the 13^{th} voting session) and group 4 (after the 29^{th} voting session) virtually finish their game because they did not reach the majority under the 3rd rounds condition. This means that they received a reduced the payoff accordingly with the instructions of the experiment. It is also interesting to notice that the steady state lock in rule T reached by group 2 cost a double “virtual” interruption of the game after round 13 and 22.

An even clearer picture of the different voting pattern followed by the 4 groups of the experiment 2 is given by figs. 3.1, 3.2, 3.3 and 3.4. The larger are the cycles shown in the figures the higher is the cyclicity of the voting behavior.

Tab. 3.2 Results from the second experiment

Group 1

	Rules to be voted																										
Players	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**
N	1	0	0	1	1	0	1	1	0	1	1	0	1	1	0	0	1	0	0	1	0	1	1	0	0	1	0
NE	1	0	1	1	1	1	0	0	0	1	1	1	0	1	0	0	1	1	0	0	1	0	0	1	0	0	1
SE	0	0	0	1	1	1	0	1	1	0	1	1	0	1	1	0	1	1	0	1	1	0	1	1	0	1	1
SO	1	0	1	1	0	0	1	0	0	1	0	1	1	0	1	1	0	0	1	0	0	1	0	0	1	0	0
NO	0	1	1	0	1	1	1	0	0	1	0	0	1	1	1	1	0	1	1	0	1	1	0	1	1	0	1
accepted	T	-	E	T	I	E	T	-	-	T	I	E	T	I	E	-	I	E	-	-	E	T	-	E	-	-	E

Group 2

	Rules to be voted																																																			
Players	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**
N	1	1	0	0	1	1	1	1	0	1	1	0	0	1	0	0	1	0	1	1	0	0	1	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	1	0	1
NE	1	1	1	1	0	1	1	0	1	0	1	1	0	1	1	1	1	1	0	1	1	0	1	1	0	1	1	0	1	1	0	1	1	0	1	1	0	1	1	0	1	1	0	1	1	0	1	1	0	0	1	0
SE	1	0	0	1	0	0	1	0	1	1	0	1	0	0	1	1	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0
SO	0	1	0	1	1	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
NO	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	1	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
accepted	T	I	-	T	-	-	T	-	-	T	-	-	-	-	-	T	-	-	T	-	-	-	-	E	T	-	-	T	-	-	T	-	-	T	-	-	T	-	-	T	-	-	T	-	-	T	-	-	T	-	-	T

Group 3

	Rules to be voted																																						
Players	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**
N	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	1	0	1	1	0	1	1	1	0	1	1	0	1	1	0	1	1	0	1	1
NE	1	0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	1	1	0	0	1	0	0	1	0	0	1
SE	1	0	0	1	0	1	1	0	0	1	0	0	1	0	0	1	0	1	1	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
SO	1	0	1	1	0	1	1	0	0	1	0	0	1	0	1	1	0	1	1	0	0	1	0	0	1	1	0	1	0	1	0	0	1	1	0	1	0	0	1
NO	0	0	1	0	0	1	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	1	1	0	1	0	0	1	0	0	1	0	0	1	1	0	1	0	0
accepted	T	-	-	T	-	E	T	-	-	T	-	-	T	-	-	T	-	E	T	-	-	T	I	-	T	I	E	-	-	E	-	-	E	-	-	E	-	-	E

Group 4

	Rules to be voted																																												
Players	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**	**T**	**I**	**E**
N	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	1	0	0	1	0	0	1	1	0	0	0	0	1	0	1	0	0	1	1	0	0	1	0	0	1	0	1
NE	1	0	1	0	0	1	1	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	1	0	1	0	0	1	0	0	1	0	0	1	0	1	1
SE	1	1	0	0	1	1	0	1	1	0	0	1	0	0	1	0	0	1	0	1	1	0	1	1	0	0	1	0	1	1	0	1	1	0	1	1	0	1	1	0	1	1	0	1	1
SO	1	0	0	1	0	0	1	0	0	1	1	1	1	0	0	1	1	1	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
NO	1	0	1	1	1	0	1	0	0	1	0	0	1	0	1	1	0	1	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	0	0	0	1	0	0	1	0	0	1	0	0
accepted	T	-	E	-	-	E	T	-	E	-	-	E	-	-	E	-	-	E	T	-	-	T	-	-	T	-	-	-	-	-	T	-	E	-	-	E	T	-	-	T	-	-	T	-	E

A way to look for an answer to the unexpected results obtained from the experiments is searching for individual error made by the participants. To make an individual error means to vote for a rule which is less efficient in terms of payoff than another one. The definition of efficient voting behaviour here used follows closely what just seen in the introductory section of this work. The cyclical voting pattern expected accordingly with the Condorcet paradox requires that the voters always vote for their first best alternative as well as for their second best alternative while never vote for the third best one. To compute an index which is coherent with this specific definition of efficiency, means to weight in the same way a positive voting for the first and the second choice and a negative vote for the third choice.

Tables 3.3, 3.3bis, 3.4 and 3.4bis show the percentages of efficient voting choices for each player of each group. More precisely, tables 3.3 and 3.4 report the efficiency percentage for each rule (respectively for experiment 1 and 2) while tables 3.3bis and 3.4bis account for the mean efficiency of voting. Looking to table 3.3 and 3.4 one can note that some players have a very low efficiency performance (the extreme case of wrong behaviour is given by player SE from group 1 of experiment 1 who had a zero efficiency score, i.e. s/he has always mistaken). In both the experiments none of the players achieved a total efficiency score. The player who got the highest score (83,33) in tab.3.3bis was SO of group 1 while in tab. 3.4bis the maximum efficiency score (69,23) was obtained by player NE of group 3.

From a general point of view the subjects of the experiment 1 have been more efficient than the participants of the experiments 2, but this is due to the fact that experiment 1 lasted less than experiment 2. Furthermore it is worth underlining that the definition of efficiency that we have used so far is referred to the one shot solution of the game. This means that a dynamical pattern like the one produced for example by group 2,3, and 4 of experiment 1 is equal to a perfectly efficient voting process, while a stable pattern like the one followed by group 2 of experiment 2 is, by definition, very near to a perfect inefficient voting process.

After experiment 2 the participants were invited to answer to some questions regarding

Fig. 3.1 Vote series in exp2

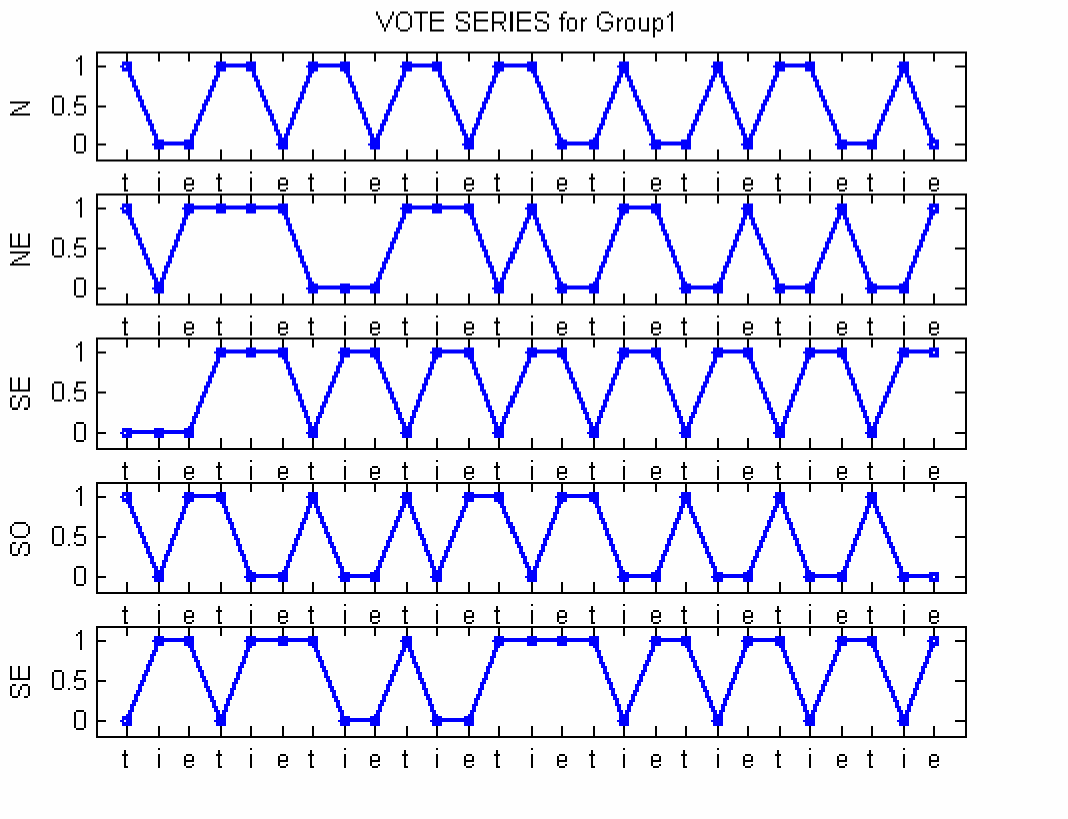


Fig. 3.2 Vote series in exp2

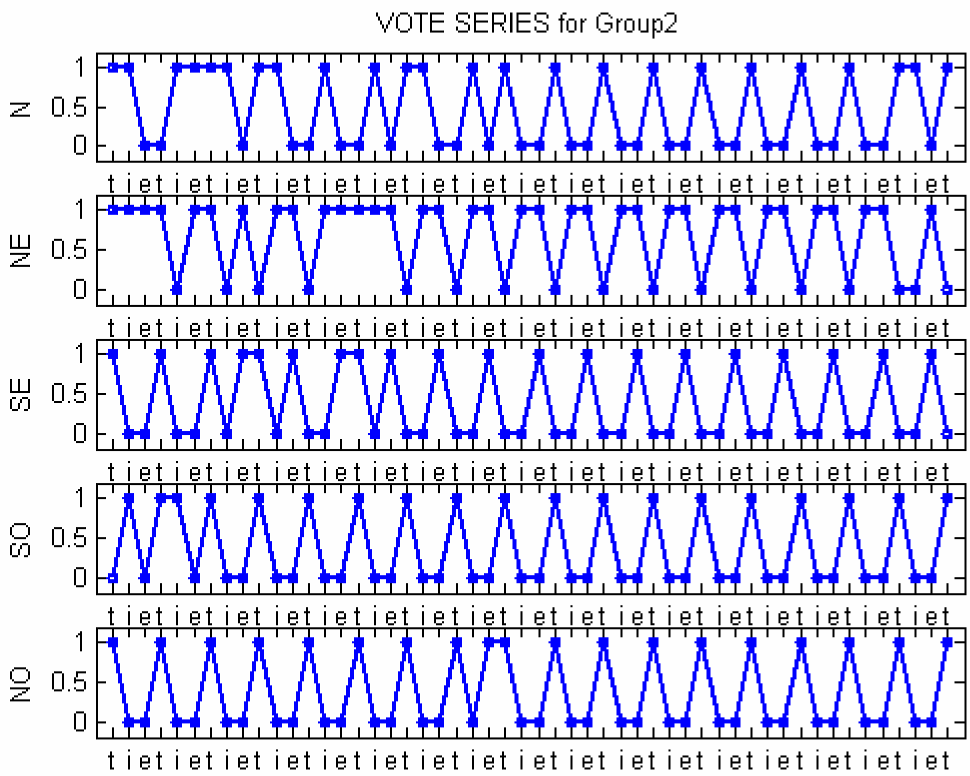

VOTE SERIES for Group2
N
NE
SE
SO
NO
1
0.5
0
t i e t i e t i e t i e t i e t i e t i e t i e t i e t i e t i e t i e t i e t i e t i e t i e t i e t

Fig. 3.3 Vote series in exp2

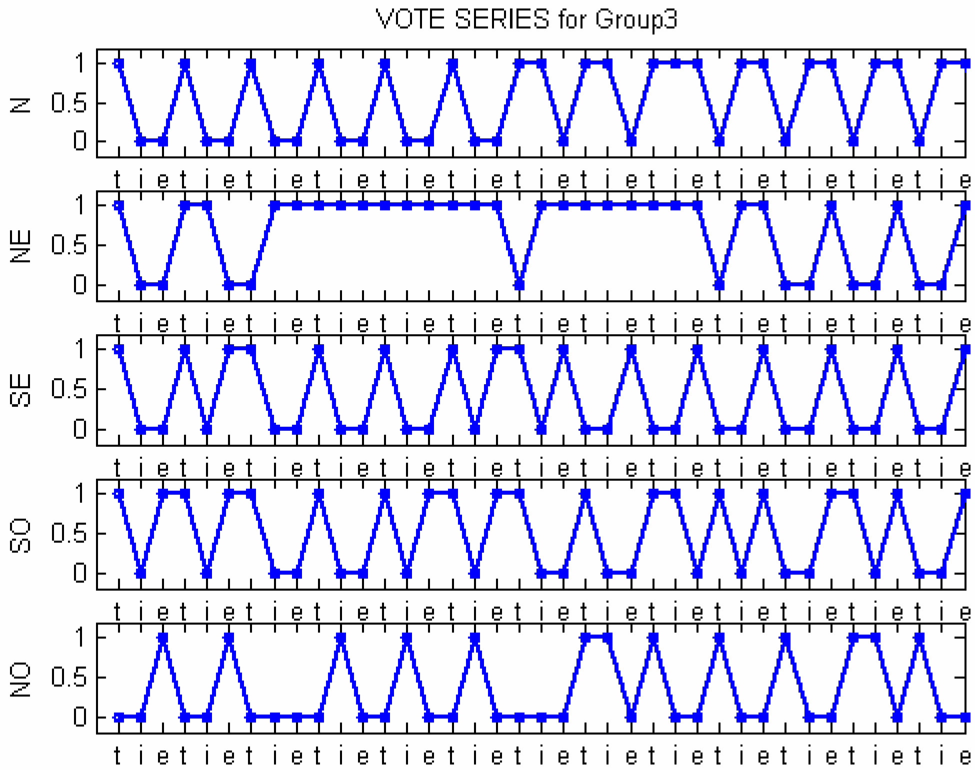
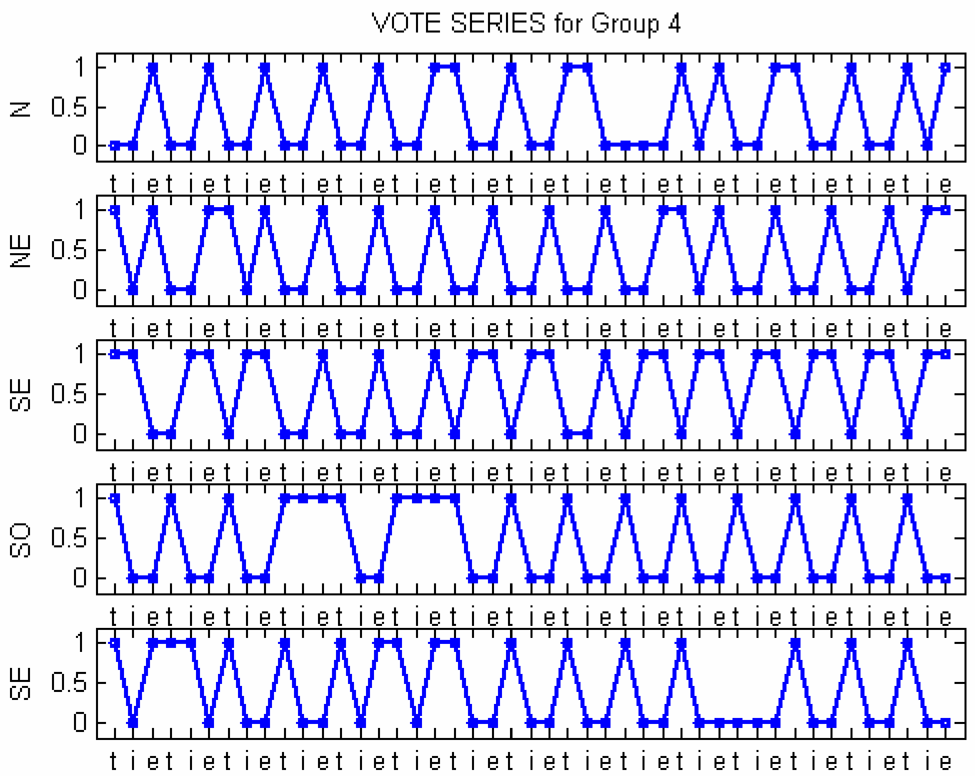


Fig. 3.4 Vote series in exp2



Tab. 3.3 Efficiency of vote (exp1)

Role	Group	Vote% T	Vote% I	Vote% E
N	1	0,00	100,00	25,00
NE	1	60,00	0,00	50,00
SE	1	0,00	0,00	0,00
SO	1	100,00	50,00	100,00
NO	1	100,00	75,00	50,00
N	2	100,00	100,00	0,00
NE	2	80,00	0,00	75,00
SE	2	80,00	0,00	100,00
SO	2	80,00	75,00	25,00
NO	2	100,00	100,00	25,00
N	3	100,00	20,00	20,00
NE	3	100,00	40,00	80,00
SE	3	20,00	0,00	100,00
SO	3	100,00	60,00	0,00
NO	3	80,00	100,00	20,00
N	4	80,00	60,00	0,00
NE	4	80,00	20,00	100,00
SE	4	80,00	0,00	100,00
SO	4	100,00	80,00	0,00
NO	4	100,00	100,00	0,00

Tab. 3.3bis Mean efficiency of vote (exp1)

Role	Group	Vote %
N	1	41,67
NE	1	36,67
SE	1	0,00
SO	1	83,33
NO	1	75,00
N	2	66,67
NE	2	51,67
SE	2	60,00
SO	2	60,00
NO	2	75,00
N	3	46,67
NE	3	73,33
SE	3	40,00
SO	3	53,33
NO	3	66,67
N	4	46,67
NE	4	66,67
SE	4	60,00
SO	4	60,00
NO	4	66,67

Tab. 3.4 Efficiency of vote and ranking of rule achieved by survey (exp2)

Role	Group	Vote% T	Vote% I	Vote% E
N	1	66,67	88,89	0,00
NE	1	33,33	44,44	77,78
SE	1	11,11	88,89	88,89
SO	1	100,00	0,00	33,33
NO	1	77,78	33,33	77,78
N	2	77,78	52,94	5,88
NE	2	22,22	82,35	100,00
SE	2	27,78	0,00	88,24
SO	2	94,44	11,76	0,00
NO	2	100,00	0,00	5,88
N	3	69,23	53,85	38,46
NE	3	53,85	69,23	84,62
SE	3	53,85	0,00	69,23
SO	3	84,62	7,69	61,54
NO	3	46,15	38,46	15,38
N	4	46,67	6,67	60,00
NE	4	20,00	6,67	100,00
SE	4	6,67	73,33	93,33
SO	4	100,00	13,33	13,33
NO	4	93,33	6,67	20,00

Role	Group	Rank T	Rank I	Rank E
N	1	**3**	**1**	**2(+)**
NE	1	**3**	**2(-)**	**1**
SE	1	**3**	**2(+)**	**1**
SO	1	**1**	**3**	**2(-)**
NO	1	**1**	**3**	**2(=)**
N	2	**3**	**1**	**2(+)**
NE	2	**3**	**2(-)**	**1**
SE	2	**3**	**2(+)**	**1**
SO	2	**1**	**2(=)**	**3**
NO	2	**1**	**2(+)**	**3**
N	3	**3**	**1**	**1**
NE	3	**3**	**2(-)**	**1**
SE	3	**2(+)**	**3**	**1**
SO	3	**2(+)**	**3**	**1**
NO	3	**1**	**2(-)**	**3**
N	4	**2(+)**	**3**	**1**
NE	4	**3**	**2(-)**	**1**
SE	4	**3**	**2(+)**	**1**
SO	4	**1**	**3**	**2(-)**
NO	4	**1**	**3**	**2(-)**

Tab. 3.4bis Mean efficiency of vote (exp2)

Role	Group	Vote %
N	1	51,85
NE	1	51,85
SE	1	62,96
SO	1	44,44
NO	1	62,96
N	2	45,53
NE	2	68,19
SE	2	38,67
SO	2	35,40
NO	2	35,29
N	3	53,85
NE	3	69,23
SE	3	41,03
SO	3	51,28
NO	3	33,33
N	4	37,78
NE	4	42,22
SE	4	57,78
SO	4	42,22
NO	4	40,00

Fig. 3.5: NORTH player vote efficency (exp. 1)

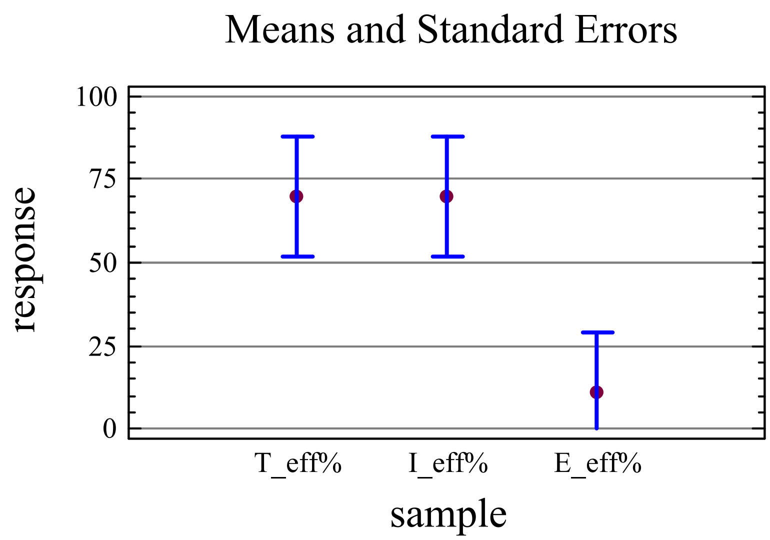


Fig. 3.6: NORTH-EAST player vote efficency (exp. 1)

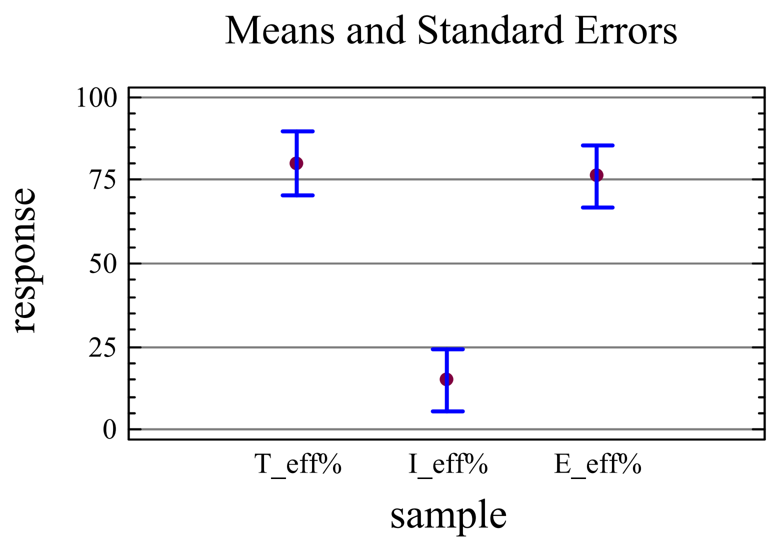


Fig. 3.7 SOUTH-EAST player vote efficency (exp. 1)

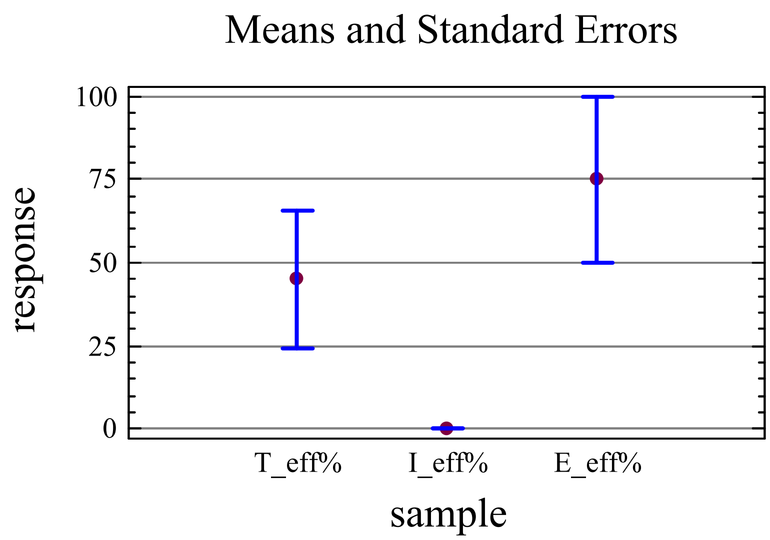


Fig. 3.8: SOUTH-WEST player vote efficency (exp. 1)

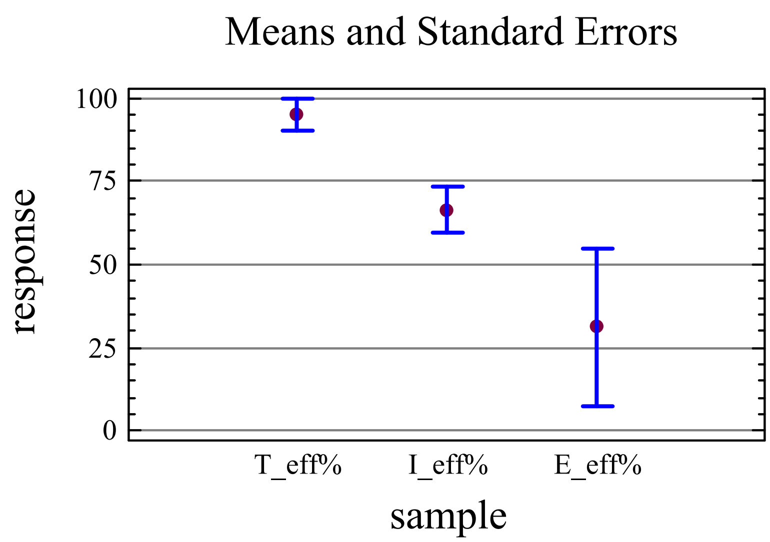


Fig. 3.9: NORTH-WEST player vote efficency (exp. 1)

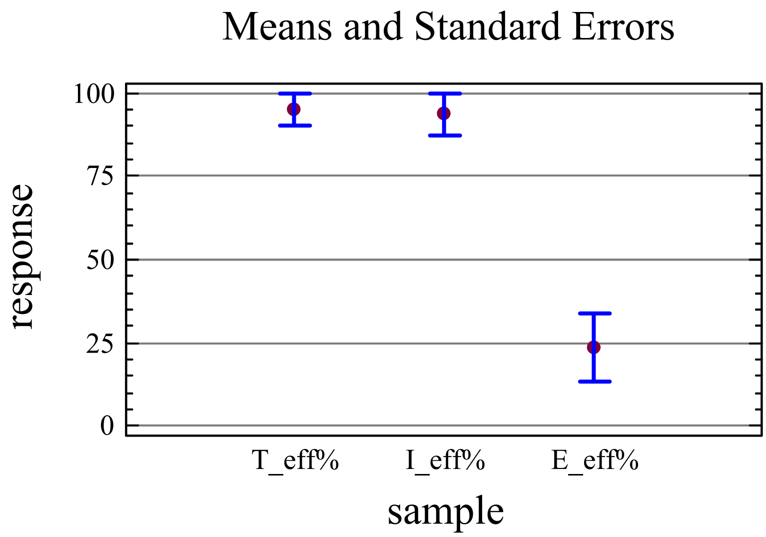


Fig. 3.10: NORTH player vote efficency (exp. 2)

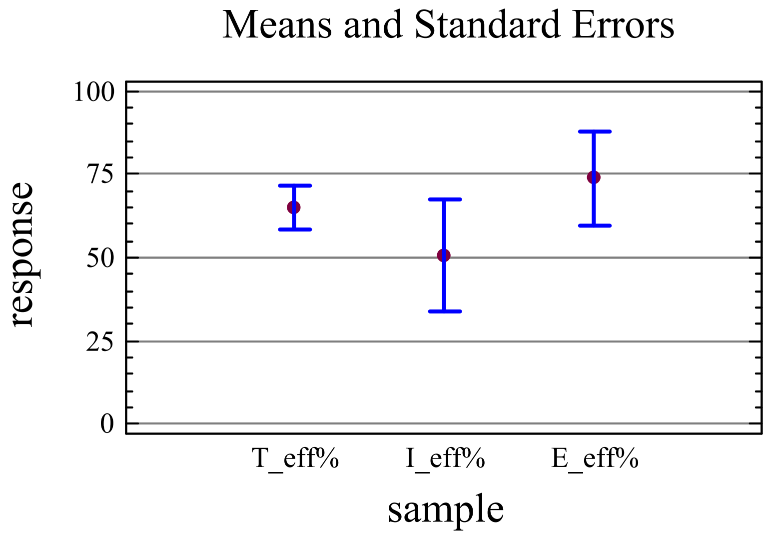


Fig. 3.11: NORTH-EAST player vote efficency (exp. 2)

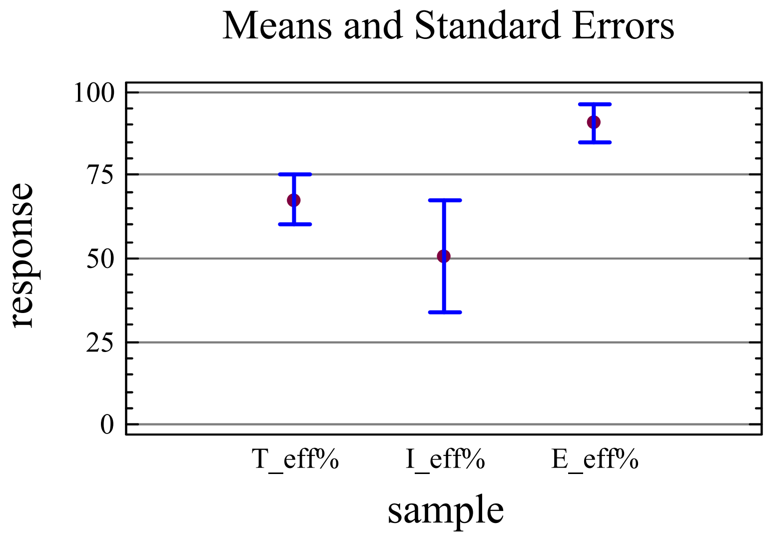


Fig. 3.12: SOUTH-EAST player vote efficency (exp. 2)

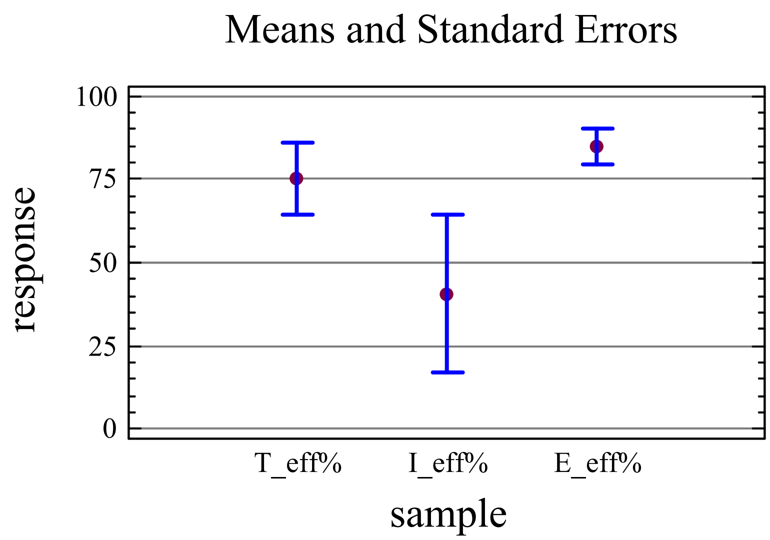


Fig. 3.13: SOUTH-WEST player vote efficency (exp. 2)

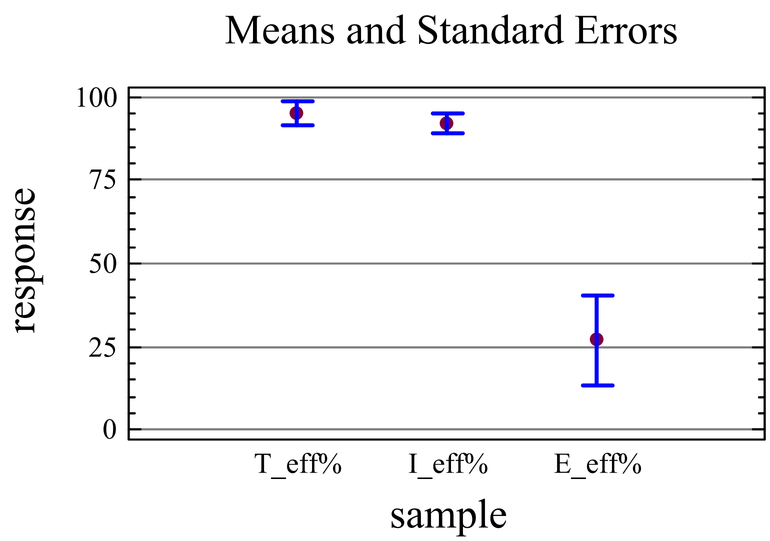
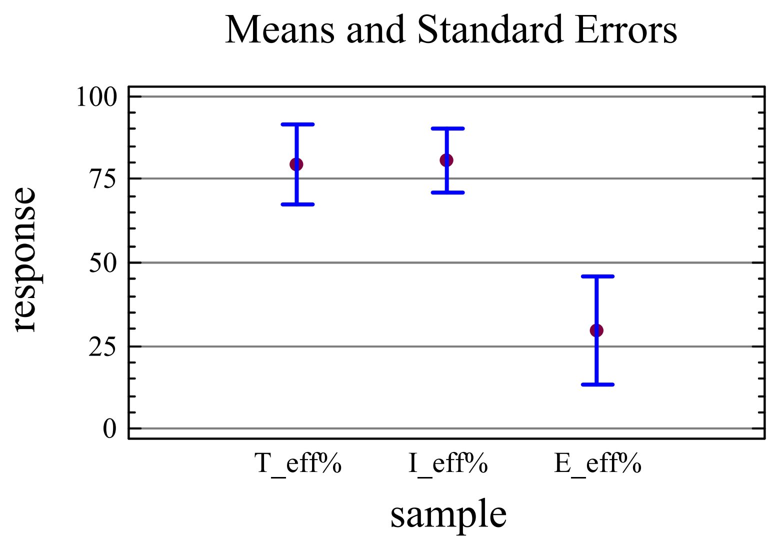


Fig. 3.14: NORTH-WEST player vote efficency (exp. 2)



Tab. 3.5 Game behaviour and rationality (exp1)

Group 1

ROUND	1	2	3	4	5	6	7	8	9
Rule Played	T	I	T	E	T	E	T	E	T
NE	m	i	e	e	i	e	m	e	m
SE	m	i	e	e	m	e	m	e	m
SO	e	i	m	e	m	e	e	e	e
NO	i	i	i	e	e	e	m	e	i
N	m	i	e	e	m	e	e	e	e

Group 2

ROUND	1	2	3	4	5	6	7	8	9
Rule Played	T	E	T	E	T	E	T	E	T
NE	i	e	e	e	e	e	e	e	e
SE	e	e	m	e	e	e	i	e	m
SO	e	e	i	e	m	e	e	e	e
NO	m	e	i	e	i	e	m	e	i
N	i	e	e	e	m	e	m	e	m

Group 3

ROUND	1	2	3	4	5	6	7	8	9
Rule Played	T	E	T	E	T	E	T	T	E
NE	e	e	m	e	e	e	e	i	e
SE	m	e	e	e	e	e	e	i	e
SO	m	e	m	e	m	e	m	e	e
NO	i	e	i	e	i	e	i	m	e
N	e	e	m	e	m	e	i	e	e

Group 4

ROUND	1	2	3	4	5	6	7	8	9
Rule Played	T	E	E	T	E	T	E	T	E
NE	e	e	e	i	e	e	e	e	e
SE	e	e	e	e	e	e	e	e	e
SO	m	e	e	e	e	i	e	m	e
NO	i	e	e	m	e	e	e	i	e
N	e	e	e	i	e	e	e	m	e

Tab. 3.6 Game behaviour and rationality (exp2)

Group 1

ROUND	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Rule Played	T	E	T	I	E	T	T	I	E	T	I	E	I	E	E	T	E	E
NE	e	e	m	i	e	m	e	i	e	e	i	e	i	e	e	i	e	e
SE	i	e	m	i	e	m	i	i	e	i	i	e	i	e	e	m	e	e
SO	e	e	e	i	e	e	e	i	e	e	i	e	i	e	e	m	e	e
NO	e	e	i	i	e	i	m	i	e	m	i	e	i	e	e	e	e	e
N	e	e	e	i	e	e	e	i	e	e	i	e	i	e	e	e	e	e

Group 2

ROUND	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Rule Played	T	I	T	T	T	T	T	E	T	T	T	T	T	T	T	T	T	T
NE	e	i	m	e	e	e	m	e	e	e	i	e	i	e	e	m	e	e
SE	e	i	e	e	e	m	e	e	e	e	i	m	i	m	e	m	i	i
SO	e	i	e	m	m	e	e	e	m	m	e	e	e	e	m	e	e	e
NO	m	i	i	i	i	i	i	e	i	i	m	i	m	i	i	i	m	m
N	e	i	m	m	i	i	i	e	m	m	e	m	e	m	i	e	m	m

Group 3

ROUND	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Rule Played	T	T	E	T	T	T	T	E	T	T	I	T	I	E	E	E	E	E
NE	e	e	e	m	e	i	m	e	m	e	i	e	i	e	e	e	e	e
SE	e	m	e	e	e	i	m	e	e	e	i	e	i	e	e	e	e	e
SO	i	e	e	e	i	m	e	e	i	m	i	m	i	e	e	e	e	e
NO	m	i	e	i	m	e	i	e	i	i	i	i	i	e	e	e	e	e
N	i	m	e	m	m	e	e	e	e	m	i	m	i	e	e	e	e	e

Group 4

ROUND	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Rule Played	T	E	E	T	E	E	E	E	T	T	T	T	E	E	T	T	T	E
NE	m	e	e	e	e	e	e	e	e	i	i	e	e	e	m	i	e	e
SE	m	e	e	e	e	e	e	e	e	i	i	m	e	e	m	i	m	e
SO	e	e	e	m	e	e	e	e	m	e	e	m	e	e	e	e	m	e
NO	i	e	e	i	e	e	e	e	i	m	m	m	e	e	i	m	i	e
N	e	e	e	m	e	e	e	e	m	e	e	e	e	e	e	e	e	e

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