Playing the wrong game: An experimental analysis of relational complexity and strategic misrepresentation[*]

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[*]This paper is an extension and revision of a previous paper entitled "Representing Others' Preferences in Mixed Motive Games: Was Schelling Right?". Financial support from the Italian Ministry of Education, University and Research (MIUR) and from the Cognitive Science Laboratory of Rovereto is gratefully acknowledged. We thank Yaakov Kareev, Marco LiCalzi, Andreas Ortmann, Burkhard Schipper, Reinhard Selten, and seminar participants in Graz, Barcelona and Prague for useful comments and suggestions on a previous draft. We thank Marco Tecilla and Ivan Soraperra for the design of the experimental software. The usual disclaimer applies.

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Abstract

It has been suggested that players often produce simplified and/or misspecified mental representations of interactive decision problems (Kreps, 1990). We submit that the relational structure of players' preferences in a game induces cognitive complexity, and may be an important driver of such simplifications. We provide a formal classification of order structures in two-person normal form games based on the two properties of *monotonicity* and *projectivity*, and present experiments in which subjects must first construct a representation of games of different relational complexity, and subsequently play the games according to their own representation. Experimental results support the hypothesis that relational complexity matters. More complex games are harder to represent, and this difficulty is correlated with measures of short term memory capacity. Furthermore, most erroneous representations are less complex than the correct ones. In addition, subjects who misrepresent the games behave consistently with such representations according to simple but rational decision criteria. This suggests that in many strategic settings individuals may act optimally on the ground of simplified and mistaken premises.

JEL classification codes: C70, C72, C91, D01

Keywords: pure motive, mixed motive, preferences, bi-orders, language, cognition, projectivity, monotonicity, short term memory, experiments

1 Introduction

While it is generally assumed that the structure of a game is well understood and common knowledge among players, some game theorists have challenged this assumption. For example, Kreps (1990) has suggested that "in choosing his actions in the short run, the individual builds a model of his choice problem, a model which is typically a simplification or a misspecification (or both) of the 'true situation"' (pp. 152). However, how do individuals simplify or misspecify the 'true situation' is still a rather unexplored issue.

An important early exception is Thomas Schelling's classic book, The strategy of conflict. Schelling reports John Strachey, the former British Defense Minister, telling him that although he had known that conflict could coexist with common interest, he had thought that the two were inherently separable, and had never considered them as part of an integrated structure (Schelling 1980, vi). Strachey's words neatly capture an important idea in Schelling's (1960) book: that representing others' strategic motivations may be a source of cognitive difficulty for players when coordination and conflict motives are intertwined in the same game.

For this purpose Schelling introduces a basic and important distinction between "pure motive" and "mixed motive" games. The former are games in which preferences of players are rank-correlated, as in the protoypical examples of pure coordination games (positive correlation) and conflict games (negative correlation). The latter games present a more complex, non correlated structure of preferences, blending coordination opportunities with antagonistic motivations. The point Schelling makes is that while pure motive games are in general easy to grasp, mixed motive games are puzzling and inherently harder to understand. He strikingly remarks that while our vocabulary is rich of words designating common interest or adversarial relationships, there are no words to designate the relation between players in a mixed motive game: while we have a rich lexicon for partners or for opponents, how to designate someone who is a partner *and* an opponent at the same time?

A similar issue has sometimes surfaced in attempts to provide game theoretic prescriptive advice to decision makers. For example, Adam Brandenburger and Barry Nalebuff's (1996) bestseller makes a central argument that managers seldom correctly identify the peculiar mix of competition and cooperation hidden in most business interactions (they feel a revealing need to fill the gap in our dictionaries, coining the hybrid word: co-opetition). Anecdotal evidence from the history of decision making also abounds; for example, Robert McNamara's (1999) recent reappraisal of "missed opportunities" during the Vietnam war provides a rich sample of episodes in which decision makers from both conflicting parties essentially failed to recognize the existence of possible cooperation within conflict and, more generally, recognizes misrepresentations - "wrong mindsets", in his words - of the nature of the ongoing interaction as a major driver of the evolution of the war.

In this paper, we experimentally address the issue of what makes it difficult for an individual to build a correct mental model of a strategic situation and, following Schelling's intuition, we start by focusing on how different payoff structures present different challenges for the individuals' cognitive ability to represent the game correctly. More specifically, we investigate the difficulty to correctly represent the relations of players' preferences over the game outcomes. For this purpose we introduce the notion of relational complexity of a two-person game representation, and we define it in terms of the structural properties of bi-orders representing the players' payoffs.

Since we want to investigate the extent to which strategic decision making is affected by misrepresentations of the underlying game structure, we proceed in two steps. First, we take a "semantic" stance, and look directly at the cognitive difficulties in representing intertwined order relations which are isomorphic to the preference structures of some classical games. We believe that this may help to disentangle representational factors from other cognitive and behavioral components, and may provide a broader perspective on the difficulties of representing interactive situations. Subsequently, we move to a classical decision making context, looking at how representation difficulties interact with actual decision making - i.e., how some puzzling behaviors may be interpreted in the light of erroneous underlying models of the game.

Our experimental results confirm the appropriateness of our classification for the purpose of understanding individual failures in representing complex relational structures. When facing games of high relational complexity, individuals tend to construct simpler representations, often of a pure-motive type. Thus, although introducing a finer classification, our results confirm that Schelling's insight was essentially correct, i.e., order relations associated to mixed motive games are significantly more difficult to represent than those mirroring pure motive games. We also show that failures in representing order structures of higher complexity are correlated with individual computational capability, as approximated by a measure of short term memory capacity.

In addition, we show that behavior of individuals misrepresenting games is consistent with their erroneous representations. In a way, such individuals play a different game, of a simpler nature. Moreover, since their misrepresentations are most of the time amenable to basic solution concepts, such subjects display behavior consistent with very simple but rational criteria such as dominance or selection of actions supporting payoff-dominant equilibria.

Section 2 of the paper shortly introduces a formal classification of bi-ordered structures, which can be applied to preference relations in two-person games. For this purpose we introduce the property of *projectivity*. Projectivity and its complement (non-projectivity) well capture, in our view, the degree of entanglement of multiple order relations, as will be better clarified in the next section. Section 3 presents an experiment on the representation of bi-ordered structures and its relation to short term memory capacity. Section 4 describes a behavioral experiment in which the representation task is embedded in actual game playing. We analyze once more representational mistakes and relate them to the interpretation of subjects' strategic behavior. Finally, section 5 discusses some implications of our results and the relationship with other streams of research in behavioral game theory.

2 Bi-orders and Preference Structures

A game is usually composed of strategies, players (including Nature), and payoffs which determine the players' preferences over the set of outcomes. Sources of cognitive difficulty for individuals may in principle arise from any of these elements alone, or, possibly, from their combination. The complexity of the strategy space is indeed an important source of constraints to players' full rationality in games (chess being the paradigmatic example: e.g., Simon and Schaeffer, 1992), as partisans of bounded rationality have often suggested, and as an abounding experimental evidence by now confirms.

Much less attention, however, has been paid so far to possible cognitive difficulties arising from the structure of preferences implied by a game [1].

This diffuse neglect notwithstanding, there is increasing evidence that players can experience serious difficulties in reasoning strategically even in games in which the action space is indeed trivial, as in very simple normal form games (e.g., Stahl and Wilson, 1994; Goeree and Holt, 2001; Devetag, Legrenzi and Warglien, 1999; Costa-Gomez, Crawford, and Broseta, 2001). Since in these games strategic complexity cannot arise from the action space, we suggest that one should look at the structure of players' preferences as an important source of difficulty for strategic thinking in such situations. After all, what distinguishes a game situation from an individual decision making task is the need to jointly take into account both one's own and the other players' preferences, and this may indeed result non trivial even in those cases in which the strategy space is not exceedingly complicated.

In what follows, we restrict our attention to simple, two-person normal form game structures. A peculiar feature of two-person strategic form games is that the outcomes of strategy profiles (i.e., the cells of the bi-matrix) constitute a bi-ordered set, as the preference order of both players is imposed on them. In order to reason strategically on the game, hence, a player must mentally represent two preference orders, her own and the other player's.

In general, bi-orders can have structures of different complexity. A useful typology of bi-orders, which originated in algebraic linguistics (Marcus, 1967; Schreider, 1975; Mel'cuk, 1988) and which is largely used in the theory of parsing, distinguishes levels of intricacy in the interrelation between two orders on the same set using the properties of *monotonicity* and *projectivity*.

[1] Ariel Rubinstein has recently shown some constraints on defining preferences in a simple propositional language: see Rubinstein (2000, ch. 4).

Before introducing a few formal definitions, an informal presentation of such properties may be useful.

A bi-order is a pair of order relations (say, $\leftarrow$ and $<$) on a set S. Let's assume for the sake of simplicity that both relations are linear orders [2]. A bi-order is monotonic if one relation preserves the order of the other (the bi-order is isotonic) or it just reverses it (the bi-order is antitonic). Projectivity can be intuitively expressed by saying that if one writes down the sequence of elements of S according to the $<$ relation, and draws the arrows directly subordinating (i.e., covering) the same elements according to $\leftarrow$, the $\leftarrow$ covering arrows should never cross each other. Finally, a bi-order is non-projective when it is not projective. Non projectivity can be intuitively expressed by saying that there is no way to arrange the sequence of elements of S according to the $<$ relationship, in such a way that the $\leftarrow$ arrows never cross each other.

Fig. 1 shows an example with four elements and two different types of arrows - continuous and dashed - representing the covering relations of $<$ and $\leftarrow$ respectively.

= Fig. 1 here =

More formally:

DEFINITION 1: Monotonic projectivity:
Let a_i, $a_j \in$ S, and let $\leftarrow$ and $<$ be two linear order relations defined on S; a doubly ordered set S is called isotonically projective if:
for i $\neq$ j $a_i < a_j$ iff $a_i \leftarrow a_j$
It is called antitonically projective if:
for i $\neq$ j $a_i < a_j$ iff $a_j \leftarrow a_i$
It is called monotonically projective if it is isotonically projective or antitonically projective.

DEFINITION 2: Projectivity:
Let a_i, a_j, $a_k \in$ S, and let $\leftarrow$ and $<$ be two linear order relations defined on S; furthermore, let $\leftarrowtail$ be the covering relation of $\leftarrow$[3]. A doubly ordered set S is called projective if one and only one of the following conditions holds:
a) (strict projectivity) for $a_i \neq a_j \neq a_k$, $a_i \leftarrowtail a_j$ and min(a_i, a_j) $< a_k <$ max(a_i, a_j) imply the relation $a_k \leftarrow a_j$.
b) (quasi-projectivity) for $a_i \neq a_j \neq a_k$, $a_i \leftarrowtail a_j$ and min(a_i, a_j) $< a_k <$ max(a_i, a_j) imply the relation $a_i \leftarrow a_k$.

[2] One can generalize definitions to non strict order relations and to the case in which one of the relations is a tree. See for example Schreider (1975).

[3] The covering relation for linear orders is usually defined as follows (Davey and Priestley, 1990). Given an ordered set A, a linear order relation $\leftarrow$ and a_i, a_j, $a_k \in A$, a_j covers a_i ($a_i \leftarrowtail a_j$) if $a_i \leftarrowtail a_j$ implies that there are no a_k such that: $a_i \leftarrow a_k \leftarrow a_i$.

DEFINITION 3: Non projectivity:
A bi-ordered set is called non projective if it is neither monotonically projective nor projective.

Since monotonic projectivity is nested into projectivity, one can naturally hypothesize a hierarchy of cognitive difficulty: monotonic projective structures are easier to represent than projective (but non monotonic) structures, which in turn are easier to represent than non projective ones. Furthermore, since antitonic projective structures require to reverse one order to obtain the other one, it is reasonable to expect that they may be (slightly) more difficult than isotonic projective structures. Linguistics provides some support to this claim in the domain of language. Language is a system that has multiple order structures simultaneously acting on it: there is the sequential order of words in a phrase, as well as many other layers of syntactical (and semantic) order. For example, to parse a phrase we must be able to recognize and process altogether such order relations. A fairly well explored example is how the linear order of words relates to the dependency order - i.e. the order induced by head-modifier relations in a sentence. A dependency A-B (where A is the governing node of a syntactical tree) is projective iff all the words between A and B are included in the sub-tree of B. For example, while "I solved only that same poignant question" is projective, "Solved only that same I poignant question" is non-projective (Schneider, 1975). Empirical analysis (see Marcus (1967, ch. 6 for a review) has shown that near 100% of natural language sentences are projective (with non-projective sentences usually confined to literary usage). More recent, extensive empirical work substantially confirms these results. For example, an analysis of the so-called "Prague dependency treebank", coding a sample of about 30,000 Chzek sentences, has found less than 2% of projective ones (Schwartz, 1998). The importance of projectivity can be understood on the ground that such property allows to introduce a proper bracketing structure into the sentence: in other words, it allows to properly decompose the sentence itself into constituents. This in turn allows to manage complex sentences in the presence of working memory constraints (the interactions between working memory constraints, complex nested sentences, and understanding performance are analyzed in Just and Carpenter, 1992).

Two-person games are bi-ordered structures: our hypothesis is that the cognitive difficulty in representing a game should depend, among other things, on the specific structure of preferences. Pure motive games are monotonically projective structures - in which the two preference relations perfectly coincide - thus they are the easiest to represent; mixed motive games can be of two types: projective ones (like for example "chicken games") or non projective ones (like for example PD's). The latter should be harder to represent, and therefore understand, than the former.

3 Representing Bi-orders: Experiment 1

3.1 Description of the experiment and discussion of results

Our central claim is that there are cognitive constraints in jointly representing multiple order relationships. These constraints seem not specific of game playing only, as the example of language suggests. Thus, we expect them to emerge in more general representational tasks. In order to test our hypothesis, we designed a simple experiment in which subjects were provided with a set of objects that could be ordered by two types of order relations and had to select a subset of them that satisfied such order relations. In semantic terms, the task consists in representing a state of affairs (a "world") that satisfies a formula built up with two order relations. In particular, we tested the hypothesis that different bi-orders induce different levels of representational difficulty. As the reader will remind, we hypothesize the following order of difficulty: non projective $\triangleright$ projective (but non monotonic) $\triangleright$ antitonically projective $\triangleright$ isotonically projective, with $\triangleright$ indicating the "more difficult than" relation.

The elements our experimental subjects had to deal with were squares which differed along the two features of SIZE and COLOR (actually, shades of grey). Squares are very familiar objects, and size and color are equally familiar order relations, henceforth we expected that no peculiar difficulties could arise in understanding the task. A set of 16 squares was shown to subjects, and their task was to select, out of this set, four squares which would satisfy simultaneously two order relations (size and color) given to them. The experiment was computerized, of the "drag and drop" type (Fig. 2 reports a sample of the computer screen). The upper part of the screen reported the 16 squares from which subjects had to select their "building blocks". Four empty cells in the bottom part of the screen were the TARGET to be filled in with squares taken from the upper part so as to satisfy the formula. Instructions (see Appendix) explained the meaning of order relations and provided examples.

Instructions also stressed the fact that the particular position of the four squares in the TARGET area of the screen did not matter, as long as the four squares satisfied the two order relations given. In order to perform the task, subjects had simply to click with the mouse on one of the squares in the table and "drag" it into one of the cells in the TARGET.

= Fig. 2 here =

Subjects were presented with the four pairs of order relations shown in fig. 1, which are order-isomorphic to the payoff structures of the four two-person strategic games shown in tables 1-4.

= Tab. 1 to 4 here =

In fact, the reader can easily check that the structure of payoffs in the coordination game with Pareto-ranked equilibria is isotonically projective. The pure conflict game corresponds to the case of antitonic projectivity. The projective case is drawn using the chicken game as a template, while the non-projective case is modelled after a Prisoner's Dilemma.

Thus, the following four pairs of order relations were presented to subjects:

- isotonic projectivity (monotonic)
  $S(W) > S(Z) > S(Y) > S(X)$
  $C(W) \to C(Z) \to C(Y) \to C(X)$
- antitonic projectivity (monotonic)
  $S(W) > S(Z) > S(Y) > S(X)$
  $C(X) \to C(Y) \to C(Z) \to C(W)$
- projectivity (non-monotonic)
  $S(X) > S(Y) > S(Z) > S(W)$
  $C(Z) \to C(Y) \to C(X) \to C(W)$
- non-projectivity
  $S(X) > S(Y) > S(Z) > S(W)$
  $C(W) \to C(Y) \to C(Z) \to C(X)$

S denotes SIZE and C denotes COLOR. The four squares are labelled X, Y, Z, and W.

A first experimental session was conducted at the University of Venice, and it involved a pool of 30 subjects who were students enrolled in a Master in Business Administration. The subjects had a monetary incentive to give correct responses in the experiment, as they were paid a fixed fee for their participation, plus an amount of 3 euros for each correct answer. The pool was divided into two sub-groups in which the order of presentation of the four bi-orders was varied, to control for learning effects.

The experiment was subsequently replicated with identical conditions at the Computable and Experimental Economics Lab of the University of Trento, using a pool of 40 undergraduate students recruited by posting ads at the various department buildings. Table 5 reports the numbers and relative frequencies of *correct* responses per task in the Venice and Trento pools respectively, distinguishing between the two sub-groups.

= Table 5 here =

Differences in the number of correct answers between the two sub-groups in each of the four tasks are not statistically significant in both the Venice and Trento pool ($p \geq .40$, Fisher exact test): therefore we can refer to the pooled data in the last column of the table. The first thing to notice by looking at aggregate results is that, notwithstanding the relatively better performance of the Trento pool in each task, the observed frequencies of correct answers in both pools suggest an order of difficulty that exactly mirrors our hypothesis: relations which are monotonically projective are relatively easy to construct, with the isotonic one easier than the antitonic. On the contrary, non-monotonic projective and non-projective bi-orders seem relatively more difficult. No statistical differences were found between the observed frequencies in the two experiments (p-values range from $.17$ to $.76$, Mann-Whitney U test), therefore from now on we refer to the pooled data reported in table 6.

= Table 6 here =

Clearly, aggregate analysis alone is not sufficiently informative in this experiment, as the single observations (performance in each task) are not independent. Hence, we performed non-parametric tests on the individual strings of successes (1) and failures (0) in the four tasks to test against the null hypothesis that successes and failures were randomly distributed.

A Cochran test performed on the four related samples allows to reject the null hypothesis that the correct answers in the tasks are equally distributed at the 1% significance level[4]. We can hence reject the null hypothesis that the four tasks presented an identical level of difficulty for our subjects.

23 subjects made no mistakes in any of the four tasks, while 2 subjects made the highest possible number (4) of mistakes. Disregarding these 25 subjects' performances as noninformative, out of the remaining 45 subjects, 35 (78%) behaved in accordance with our conjecture, i.e., they made mistakes in a way that did not violate our hypothesized hierarchy of difficulty. More specifically, 17 subjects made a mistake only in the non-projective task, 9 constructed both monotonic bi-orders correctly but made mistakes in the projective and non-projective tasks, and 9 correctly constructed only the isotonic case. Excluding the extreme cases of everything right and everything wrong, there are fourteen possible strings of 1's and 0's of length 4, of which the only three that are strictly consistent with our conjecture - assuming the sequence isotonic-antitonic-projective-nonprojective -, are the following: 1-1-1-0, 1-1-0-0-, 1-0-0-0. If all strings were equiprobable, we should expect to find approximately $3/14 \times 45 = 9.2$ strings that confirm our hypothesis, instead of the 35 that appeared in our data. The difference

[4]Cochran's Q = 52.796, $p = .000$.

between the observed and the theoretical distribution is statistically significant at the $p = .001$ level by a Chi-square test.

We subsequently made pairwise comparisons by applying a McNemar test. All differences between pairs are statistically significant (Isotonic-antitonic bi-order: $p = .002$; isotonic-non-projective bi-order: $p = .000$; isotonic-projective bi-order: $p = .000$; antitonic-non projective bi-order: $p = .022$; non projective-projective bi-order: $p = .003$; McNemar test) *except* the difference between the antitonic and the projective bi-order ($p = .096$), which is only weakly significant.

Hence, the data strongly confirm our hypothesis in all but the antitonic-projective pair, although also in this last case observed frequencies of mistakes are as expected.

Additional insight can be gained by conducting an analysis of the most common types of errors that subjects made. While mistakes in the "projective" task show a relatively high variance, mistakes in the "non-projective" task show a rather revealing pattern. In fact, of the thirty-nine subjects who did *not* answer correctly in this task, twenty (51.3%) constructed an *antitonic* bi-order, while fifteen (38.5%) constructed an *isotonic* bi-order.

Thus, as we hypothesized, individuals, out of a non-projective pair of relations, tend to simplify their representations by perceiving and extracting monotonic bi-orders.

3.2 Short-term Memory Capacity and Representation

Why should some bi-order structures be harder to represent than others? Research in the psychology of mental models (Johnson-Laird 1983) has repeatedly - although not conclusively - suggested that short -term memory constraints may hinder the individual ability to edit a complete, accurate mental representation of a given task-environment. Since the pioneering work of George Miller (1956), it is well-known that individuals can hold only a limited amount of information active in their short-term memory, which is a basic bottleneck in human information processing. Thus, complex structures may overload individual short-term memory capacity, causing incomplete, over-simplified and often mistaken representations of these structures. The load on short-term memory capacity, however, may be reduced by the ability to compress information or decompose it into smaller components.

Clearly, bi-ordered structures differ in the way information can be compressed or decomposed. For example, isotone bi-orders can be simply processed as a single order, while antitone ones can be easily obtained by reversing a single order. The case of projective and non-projective bi-orders is less trivial. However, projective structures have the property of naturally generating a proper decomposition into a tree of constituents. To see this, it suffices to bracket each pair of elements related by the covering relation $\leftarrowtail$ of $\leftarrow$. For example, exploiting the usual order of parentheses and starting from the least element of the chain ordered by $\leftarrowtail$, one obtains the bracketing shown in fig. 3 in the case of a projective bi-order:

= Fig. 3 here =

This bracketing is “proper”, meaning that parentheses are nested.

Projectivity implies that a proper bracketing always arises. This follows naturally from the property that the $\longleftarrow\!\!\!\prec$ arrows do not intersect in projective bi-orders.

On the contrary, it is easy to see that non-projective structures fail to generate such decomposition, as it is shown in fig. 4:

= Fig. 4 here =

Consequently, there are good reasons to hypothesize that the short-term memory load of editing the representation of a projective bi-order is significantly lowered by the possibility of decomposing it into a tree of constituents (which even a simple stack memory device could easily manage: see Hopcroft and Ullman, 1979). Non-projective bi-orders on the contrary do not present such a decomposability property, and thus force one to consider all elements and their order relations simultaneously.

It has been shown that individuals differ in their short-term memory capacity (Miller, 1956; Baddeley, 1990). Hence, if short-term memory capacity limitations are a source of difficulty in representing bi-orders, one should expect the performance of individuals in our experiment to be correlated with their memory capacity.

In order to test this hypothesis, we conducted a standard Wechsler digit span test for short-term memory capacity (e.g., Walsch and Betz, 1990; see also Devetag and Warglien, 2003 for a related experiment) on 38 of the 40 subjects of the Trento pool in a separate experimental session. This simple test consists in asking subjects to repeat a series of digits which are to be read by the experimenter at the rate of one digit per second. The test is conducted sequentially on two independent sets of digit series of increasing length. For each set, the test stops when the participant fails to correctly repeat a given series. The subject’s ‘score’ in each set is given by the length of the last series that was repeated correctly (so, for example, if a subject fails to correctly repeat a series of length 6, her score will be equal to 5). The subject’s final score is then given by the higher among the two scores that were achieved in the sets. Although the score needs not directly reflect the number of ‘short-term memory slots’ available to an individual, it is generally assumed that higher scores correspond to a higher short-term memory capacity.

Table 7 reports the correlation between subjects’ score in the memory test and the total number of correct responses in the representation experiment. We computed the two standard Spearman rho and Kendall tau rank-correlation tests. Both tests support our hypothesis of a significant correlation between individual short-term memory capacity and performance in the experiment.

= Table 7 here =

= Table 8 here =

Table 8 also reports the mean STM score of subjects who were successful and of those who were unsuccessful in each of the four tasks. A T-test performed on average scores in the two samples reveals statistically significant differences in the case of the conflict and chicken games, while the differences in the coordination game and the PD are not significant. A plausible explanation for these results, besides the small data sample in the coordination case, is that differences in short term memory bounds are likely to emerge more strongly in tasks of intermediate difficulty. In fact, if the task is very easy most people can solve it no matter how low their memory capacity is, and if the task is very difficult the workload is such that even people with high scores can make mistakes, as our data on the coordination game and the PD show.

This explanation is strengthened by looking at the correlation between memory scores and the number of correct responses in the coordination and conflict games only (Kendall's tau-b = .275, $p = .036$, and Spearman's Rho = .297, $p = .035$), or between memory scores and numbers of correct responses in all tasks except the PD (Kendall's tau-b = .297, $p = .025$, and Spearman's Rho = .321, $p = .025$). The values indicate that a considerable portion of the correlation reported in table 7 is preserved by restricting its calculation to performance in the monotonic and projective tasks only.

4 Experiment 2: Representation of games and links with behavior

4.1 Experimental design and implementation

Having verified the cognitive difficulty to represent different classes of bi-orders, our second experiment is aimed at assessing how these difficulties affect game playing. For this purpose, we designed an experiment in which subjects had to construct a representation of the four games depicted in tables I-IV, and subsequently choose a strategy in each game. The experiment was divided in two parts. At the beginning of the experiment, subjects were told that in the second part they would participate in four interactive decision making tasks, and in each task they would be paired with a randomly selected opponent. In each decision making task each player could choose between two available moves. The four resulting combinations of choices generated four different scenarios (called A, B, C and D), each implying different payoffs for the two players. The possible payoffs in each game could range from 1 to 4 experimental points for each player, and would be represented by squares differing in color and size. A player's payoff depended on color (the darker the square, the higher the payoff), while the other player's payoff depended in the same way on the square size. In the first part of the experiment subjects would have to visually represent the four games according to pairs of order relations like the one in experiment 1. This was done by selecting, among a set of 16 squares, the four that satisfied the relations given; hence, the first part of the experiment was essentially equivalent to experiment 1, with the difference that subjects knew that they were representing interactive decision making tasks that would be the object of the second part of the experiment, and that the color and size of the squares represented their and their opponents' payoffs in the games. Since our interest is in bi-orders, also in this case we kept a geometric, and hence purely ordinal, representation of payoffs avoiding their direct translation into numbers. For the same reason, we avoided imposing further representational structure by using devices such as game matrices, leaving instead the simple linear display of geometric figures used in experiment 1.

The instructions stated that the representational task was strictly individual and that earnings in that task were solely a function of the number of games correctly represented, whereas earnings in the second part were contingent on one's own and one's opponent choices in the games. In order to emphasize the effects of working memory constraints we introduced a time limit of 120 seconds to complete each representation, and the four games appeared to each subject in a random order to control for learning effects.

In the second part of the experiment, each subject's computer terminal displayed the four representations that she had constructed in the first part, one at a time, and the subject had to choose a move between the two available. Hence, subjects had to play the games according to the representation that they themselves had constructed, knowing they would be randomly paired with a different opponent in each game, and all this information was common knowledge. The instructions also specified that in the case a subject had not been able to complete a representation within the time limit, the representation would be constructed arbitrarily by the computer program. Furthermore, the instructions stated that the payoffs in the games were calculated according to the moves chosen by the two players *on the basis of the correct representation*. In this way, a further incentive was introduced to represent the game correctly. The games in the first part were reported as shown in fig. 6, while in the second part they were represented as shown in fig. 7. At the beginning of each game, players were randomly assigned to either COLOR or SIZE (corresponding to the usual ROW and COLUMN roles in game matrices), and the two available moves were labelled with the two symbols of *spades* and *clubs*. Subjects did not receive any form of feedback at any time regarding their performance and/or their opponents' decisions.

The experiment was run at the Computable and Experimental Economics Lab of the

University of Trento in three separate sessions and it involved a total of 68 subjects, who were all undergraduate students who had never participated in similar experiments before. Instructions were distributed at read aloud at the beginning of the experiment (see Appendix). The overall time limit for the first part was set equal to eight minutes, whereas no time constraint was imposed in the second part. There was a show up fee of 3 euros plus anything subjects could earn in the experiment. Average overall earnings were equal to 14.8 euros; the experiment lasted sixty minutes on average, including the instruction time.

4.2 Results

Table 9 reports the frequencies of correct responses in each representation task. The observed frequencies in the different games mirror very closely those in experiment 1. The Cochran test performed on the four samples is highly significant ($p = .000$). Likewise, pairwise differences measured by a McNemar test are all statistically significant (coordination-chicken game: $p = .002$; coordination-PD: $p = .000$; conflict-chicken game: $p = .029$; conflict-PD: $p = .000$; chicken-PD: $p = .032$), although the difference between the coordination and the conflict games is only weakly so ($p = .10$).

23 subjects made no mistakes in any of the four tasks, and 4 subjects made four mistakes. Hence, there is a total of 41 subjects who made from 1 to 3 mistakes, and whose performance is therefore informative. A qualitative analysis of the individuals strings of successes (1) and failures (0) reveals that 26 out of 41 strings (63%) are strictly consistent with our hypothesized hierarchy of difficulty, against the 22% that would be expected in case of randomly distributed errors (the difference is significant at the $.001$ level by a Chi-square test)[5].

We then move to an in-depth analysis of the mistaken representations, which we restrict to the chicken game and to the Prisoner's Dilemma, because the number of errors in these two tasks is sufficiently high.

Our conjecture is that the erroneously reconstructed matrices should diverge from the correct ones in terms of their relational structure, and in particular that they should reflect the difficulty to represent complex bi-orders.

Table 10 reports the empirical distribution of mistaken games classified by type of bi-order. A useful benchmark is the distribution of possible classes of bi-orders in $2X2$ matrices. It turns out that, out of the $4! = 24$ possible $2X2$ matrices that one can build, only 1 is isotonic, only 1 is antitonic, 14 are projective and 8 are non-projective. In both the chicken and the PD case, the distribution of types of bi-orders generated by our subjects significantly differs from such distribution ($p = .0003$ for the PD case,

[5]This analysis, as well as the analogous analysis in experiment 1, would give the same results if we included the strings of subjects who did everything right and those of the subjects who made four mistakes. In fact, although these cases are not informative, they are not, as such, in contrast with our hypothesized hierarchy of difficulty.

and $p = .02$ for the chicken case, according to a Chi-squared test with simulated p-value, 1,000,000 replicates). Furthermore, according to the same test, the "erroneous PD" distribution is significantly different from the "erroneous chicken" one ($p = .002$). In other words, mistakes in the PD and in the chicken game are drawn from different distributions.

Thus, representational errors are not random. In both cases, they show more isotonic/antitonic instances and less projective/non projective ones than randomly expected ($p < .01$, one-tailed Fisher exact test). At the same time, mistakes seem to reflect the structure of the underlying representation problem: there are more projective than non-projective mistaken representations in the chicken game, and the reverse holds in the case of PD ($p = .04$, one-tailed Fisher exact test). Moreover, the chicken game is more often associated with isotonic representations than with antitonic ones, while the reverse is true for the PD game ($p = 0.01$, one-tailed Fisher exact test). Thus, the chicken game is often transformed in a coordination game, while the PD game is often transformed in a pure conflict game, although sometimes it still is transformed in a coordination game.

An important issue is how representations interact with actual choice behavior. As far as subjects that represent the games correctly are concerned, choices in the four games conform to well-established behavioral principles (see table 11). In the coordination game, subjects predominantly play the strategy that supports the payoff dominant equilibrium, although, especially when the two strategies have the same payoff sum, as in the case of row players, many play the alternative, safer strategy. In the conflict game the distribution of behavior is different from the mixed strategy equilibrium probabilities, due to a large extent to the fact that subjects take into consideration the weight of out-of-equilibrium payoffs, as related experiments have shown (e.g., Goeree and Holt, 2001). In the chicken game, the safer deferential action is more often played than the more risky and aggressive one. Finally, in the PD about 2/3 of players defect, but another third is cooperating.

Looking at the behavior of those who have misrepresented the game is more informative. We have suggested that, in general, when misrepresenting a game subjects tend to generate representations which are structurally simpler than the "correct" one. Often these representations also imply simple solution criteria. As a result, although subjects often construct the wrong game representation, they act quite rationally in the light of such erroneous constructs.

For example, 11 out of 17 subjects who have misrepresented the chicken game have reconstructed it as a game in which a single strategy profile yields the maximum payoff for both players - and 9 out of these 11 subjects have played the strategy corresponding to such profile. Of those 11 representations, 8 reconstructed the chicken game as a coordination game with a Pareto-dominant equilibrium - and in 6 out of these 8 representations subjects have played the strategy corresponding to that equilibrium. 5 of the 6 representations that do not have a "common max" profile have a dominant action -

and once more 4 out of 5 subjects play according to dominance.

Similarly, 21 out of 23 subjects who have misrepresented the PD game have introduced a dominant strategy in their representation, and have acted almost always (18 out of 21) according to dominance. Notice, however, that the dominant strategy in the erroneous representations of the PD game does not necessarily correspond to defection: 8 players represent "cooperate" as the dominant action (and 6 play according to it). From a different angle: 8 out of 23 subjects who have misrepresented the PD game have played "cooperate" (i.e., have picked the strategy that is labelled as cooperation in the correct game) and 7 of these 8 subjects have done it according to a principle of dominance or Nash equilibrium. If these apparent cooperators are more rational than one might have guessed, "defectors" with erroneous representations are not really defecting in the game they have reconstructed: indeed, all but one of them have built a game in which the equilibrium in dominant strategies for both players is also the social optimum. In summary, no matter whether they cooperate or defect according to the labels that apply in the "true" game, most players that erroneously reconstructed the PD represented it in such a way that the social optimum is also an equilibrium.

5 Discussion and Conclusion

Our experiments provide support to the view that not all normal form games are the "same", and that structural complexity matters; we suggest that besides the strategy space, relational structure is a further source of cognitive difficulty, providing a finer classification of two-person games. "Pure motive" games (i.e. monotonic payoff structures) are easier to represent, and even in the presence of "mixed motive" games, such simpler structures act as irresistible templates of interaction. We also show that a further classification, involving the property of projectivity, is useful in defining levels of relational complexity.

We also show that the same ranking of representational difficulty applies when bi-orders are explicitly embedded in the presentation of a game. Subjects aware that they are setting the stage for interactive decisions experience the same increasing difficulty as the payoff structure goes from isotonicity to non-projectivity. The analysis of the behavior of subjects that misrepresent the games reveals a few relevant features. 1) Subjects tend to simplify representations, constructing models of the game that lower its relational complexity. 2) In doing so, they still anchor to some structural features of the "true" game. For example, in the PD, subjects still generate more non-projective erroneous representations than in the chicken game. Furthermore, other features of the correct game are preserved, although probably as a bi-product of other representational processes; for example, dominance is preserved in most erroneous PD's, whereas multiplicity of equilibria emerges in many erroneous chicken games. Given such simplified representations, actual choice behavior follows simple but quite "rational" decision criteria, such as dominance or selection of actions implementing payoff-dominant equilibria. These results suggest a fresh reinterpretation of behavior in well known experimental games. We have shown that when subjects misrepresent the PD game, they tend to eliminate its social dilemma nature, in a way that both the "cooperation" and "defection" moves actually correspond to strategies that support perceived social optima.

Similarly, behavior in the chicken game can be sometimes interpreted as the result of its subjective reframing as a coordination game with a payoff-dominant equilibrium.

Other well-documented behaviors find a natural interpretation in this framework. For example, a large literature in negotiation research has pointed out that bargainers are often subjects to the "mythical fixed pie bias" (Bazerman, 1983), which causes them to wrongly assume that their interests are diametrically opposed. "The assumption of a fixed pie is rooted in social norms that lead us to interpret most competitive situations as win-lose [...]. Humans tend to generalize from these objective win-lose situations to situations that are not necessarily win-lose" (Bazerman, Baron and Shonk, 2001, p. 13). Within our framework, the fixed pie bias is almost literally the transformation of a complex bargaining game into a simpler, antitonic, antagonistic representation. Some further evidence in this sense comes from a recent experiment on complex multi-issue negotiations (Hyder, Prietula, and Weingart, 2000). The authors of the study observe that negotiators rarely achieve a Pareto-optimal solution to a given negotiation problem, and they argue that the reason lies in their incorrect 'default' representation of the situation as a zero-sum game. In fact, representing the game as zero-sum would trigger the almost exclusive use of *distributive* negotiation tactics (i.e., tactics aimed at achieving unilateral concessions from the other party) at the expense of *integrative* tactics, which would instead facilitate the achievement of agreements resulting in gains for *both* parties involved. Hence, the use of specific behavioral strategies conducting to sub-optimal agreements seems to derive, according to the authors, by an original failure of players to represent the mutual gain area in the space of solution points.

In addition, our results point to an important source of heterogeneity in a population of game players, namely differences in the relative ability to correctly represent the structure of strategic interaction. These differences make the pair with observed differences in the depth of iterated thinking in games (e.g., Nagel, 1995, Camerer, 2003, ch. 5; Camerer, Ho and Chong, 2002). Interestingly enough, both types of heterogeneity appear to be (weakly) correlated with differences in short term memory capacity, a measurable psychological proxy of individual computational capability (Newell and Simon, 1972; Devetag and Warglien, 2003; Kaareev, 1992). These observations may also help understand how players transfer behavior from previously experienced games to new ones. Knez and Camerer (2000) show that after playing a common interest coordination game, individuals are more prone to cooperate in a PD. We suggest that the coordination game may act as a template for the cognitive simplification of the subsequent PD. In fact, our taxonomy may help to predict the direction of transfer phenomena from simpler to more complex games.

Furthermore, we submit that monotonic structures may indeed be the prevailing templates of bi-orders available in memory. In a classroom experiment, students asked to provide examples of four arbitrary objects satisfying simultaneously two arbitrary order relations of the kind depicted in fig. 1 had no difficulties in finding examples for monotonic bi-orders (such as “richer is happier” or “larger towns are less healthy”); but they found it almost impossible to provide examples for non-projective bi-orders. This point reinforces the result that in our experiment subjects unable to provide a solution to the non-projective case resorted to monotonic orderings of the squares. Returning to games, it also suggests that in incomplete information games “friends” and “enemies” may be the most natural player types.

Our results may also provide some complementary cognitive ground to Ariel Rubinstein’s (1996) argument on the prevalence of linear order structures in discourse. Rubinstein claims that linear orders have some efficiency properties (in indicating an element out of a set, in being informative about a relation on a set, in minimizing the number of examples necessary to describe a relation) that justify the higher frequency with which these structures appear in natural language. Clearly, one can construct a structure-preserving map from a monotonic bi-order to a linear order, either directly (as in the isotonic case) or with an intermediate step by reversing one of the two order relations (as in the antitonic case). The same cognitive constraints that make monotonic bi-orders easier to represent may underlie the prevalence of linear orders in natural language. Projectivity is a more complex case: no simple way to reduce it to a single linear order can be found. Yet, projectivity can be thought of as a kind of compatibility between order relations, simplifying the task of managing bi-orders in short term memory. The relevance of the projectivity property in natural language suggests that further connections with Rubinstein’s argument are worth seeking.

Finally, our results provide a first, albeit partial, answer to what Colin Camerer (2003) has placed among the top ten open research questions in behavioral game theory, namely: “What game do people think they are playing?”(p. 474). Experimental game theory has so far relied on the implicit assumption that the game subjects played was the one provided by the experimenter. Our data suggest that this assumption may be misleading, and that, more generally, individuals may indeed apply optimal decision criteria to a misspecified strategic setting.

References

[1] Baddeley, A.D. (1990). *Human Memory: Theory and Practice*. Hillsdale, NJ: Erlbaum Associates.

[2] Bazerman, M. H. (1983). “Negotiator judgment: A critical look at the rationality assumption”, *American Behavioral Scientist*, 27, 618-634.

[3] Bazerman, M. H., Baron, J. and Shonk, K. (2001). “Enlarging the Societal Pie - A Cognitive Perspective”, Harvard Business School Working Paper No. 02-081.

[4] Brandenburger, A.M., and Nalebuff, J. N. (1996). *Co-opetition*. New York, NY: Doubleday.

[5] Camerer C. (2003). *Behavioral Game Theory: Experiments on Strategic Interaction*, Princeton University Press, Princeton, NJ.

[6] Camerer, C., Ho, T. and Chong, J.K. (2002). “A cognitive hierarchy theory of one-shot games: Some preliminary results”, working paper, California Institute of Technology.

[7] Costa-Gomez, M., Crawford, V., and Boseta, B. (2001). “Cognition and Behavior in Normal-form Games: An Experimental Study”, *Econometrica* **69**, 1193-1235.

[8] Davey B.A., and Priestley, H. A. (1990). *Introduction to lattices and order*. Cambridge, UK: Cambridge Univ. Press.

[9] Devetag, M.G., Legrenzi, P., and Warglien, M. (1999). “Focusing in naive game playing”, in *Deductive reasoning and strategies*, (W. Schaeken, G. De Vooght, A. Vandierendonck, and G. d’Ydewalle, Eds.), pp. 287-299. NJ: Erlbaum Associates.

[10] Devetag, G., and Warglien, M. (2003). “Games and Phone Numbers: DO Short Term Memory Bounds Affect Strategic Behavior?”, *Journal of Economic Psychology*, 24, 189-202.

[11] Goeree, J.K., and Holt, C.A. (2001). “Ten Little Treasures of Game Theory and Ten Intuitive Contradictions”, *American Economic Review*, 91, 1402-1422.

[12] Hopcroft, J.E., and Ullman, J. D. (1979). *Introduction to automata theory, languages and computation*. Reading (Mass): Addison Wesley.

[13] Hyder, E.B., Prietula, M. J., and Weingart, L. R. (2000). “Getting to Best: Efficiency versus Optimality in Negotiation”, *Cognitive Science* **24(2)**, 169-204.

[14] Johnson-Laird, P.N. (1983). *Mental Models*. Cambridge, UK: Cambridge Univ. Press.

[15] Just, M.A. and Carpenter O.A. (1992). “A Capacity Theory of Comprehension”, *Psychological Review* 99, 122-49.

[16] Kareev, Y. (1992). “Not that bad after all: Generation of random sequences”,*Journal of Experimental Psychology: Human Perception and Performance* **18**, 1189-1194.

[17] Knez, M., & Camerer, C. (2000). “Increasing Cooperation in Prisoner’s Dilemmas by Establishing a Precedent of Efficiency in Coordination Games”, *Organizational Behavior and Human Decision Processes*, 82, 2, 194-216.

[18] Kreps, D. (1990). *Game Theory and Economic Modeling*, Oxford, Oxford University Press.

[19] Marcus, S. (1967). *Algebraic Linguistics: Analytical Models*. NY: Academic Press.

[20] McNamara, R. (1999). *Argument without an end*. Public Affairs.

[21] Mel’cuk, I. A. (1987). *Dependency Syntax: Theory and Practice.*. Albany, NY: State University of New York Press.

[22] Miller, G. (1956). “ The magical number seven, plus or minus two. Some limits of our capacity for processing information”, *Psychological Review* **63**, 81-97

[23] Nagel, R. (1995). “Experimental Results on Interactive Competitive Guessing”, *American Economic Review*, 85, 1313-1326.

[24] Newell A., and Simon H.A. (1972). *Human Problem Solving*. Englewood Cliffs (NJ): Prentice Hall.

[25] Rubinstein, A. (1996). “Why are certain properties of binary relations relatively more common in natural language?”, *Econometrica* **64**, 343-356.

[26] Rubinstein, A. (2000). *Economics and Language*. Cambridge, UK: Cambridge Univ. Press.

[27] Schelling, T. (1980). *The Strategy of Conflict*, Second Edition. Cambridge, Mass.: Harvard Univ. Press.

[28] Schneider, J. A. (1975). *Equality, resemblance and order*. Moscow, USSR: MIR Publishers.

[29] Schwartz, O. (1998). “Structured Language Model Parsing”. In: *Core Natural Language Processing Technology Applicable to Multiple Languages: Final Report*, CLSP, Johns Hopkins University, 1998.

[30] Simon H.A. and Schaeffer, J. (1992). “The Game of Chess”, in *Handbook of Game Theory with Economic Applications, vol. 1*, (R. J. Aumann and S. Hart, Eds.), pp. 1-17. The Netherlands: Elsevier Science Publishers.

[31] Stahl, D.O., and Wilson, P. W. (1994). “Experimental Evidence On Players’ Models of Other Players”, *Journal Econ. Behav. Organ.* **25**, 309-327.

[32] Walsh, W.B., and Betz, N. E. (1990). *Tests and Assessment*. Englewood Cliffs, NJ: Prentice Hall.

Appendix: Instructions[6]

Instructions for experiment 1

In the following experiment you will be asked to answer some questions regarding order relations between elements. An order relation, as the term itself indicates, allows some elements of a set to be ordered according to a certain characteristic.

For example, an order relation can be defined according to SIZE: given a set of rectangles, I can say whether rectangle X is bigger, smaller or equal to rectangle Y; I can also order a set of rectangles from the smallest to the biggest, and vice-versa.

In the following experiment we will ask you to respond some questions about objects according to which two order relations can be defined: one on the basis of SIZE, the other on the basis of COLOR.

We will use the canonical symbols of order relations:

$$> \quad < \quad =$$

In the case of SIZE, the meaning of the three symbols is obvious and intuitive. For example, the expression $X > Y$ indicates that element X is bigger than element Y.

In the case of COLOR, you will be proposed four colors: black, white, and two variations of gray. It will be set by convention that the symbol $>$ means "darker than".

In the following tasks, SIZE will be indicated by the letter S, and COLOR by the letter C. Two objects can be defined according to both characteristics. For example, in the following case

= Fig. 5 here =

the left circle (X) is bigger than the right circle (Y), but the right circle is darker than the left circle. This double order relation will be expressed in the following way: $S(X) > S(Y)$, and $C(X) < C(Y)$.
Obviously, saying that $S(X) > S(Y)$ is equivalent to saying that $S(Y) < S(X)$. Therefore, the two notations will be used interchangeably.

In the experiment you will be presented four different pairs of order relations with regard to SIZE and COLOR, with each pair being defined over four elements (squares). The four squares will always be indicated with the letters X, Y, W and Z, while color and size will be denoted with C and S.

[6]The following are English translations of the instructions used in the two experiments. Original instructions in Italian are available from the authors upon request.

The task will be computerized. Your computer screen will visualize a set of **16** squares of different colors and sizes, and four empty cells. You will have to fill in the four empty cells with 4 squares (chosen out of the 16) which, according to you, satisfy the pair of order relations that will be provided to you. In order to accomplish the task, you will simply have to click with your mouse on the chosen squares and drag them to the empty cells in the TARGET. For each correct answer, you will be assigned 50 points, which will be converted in cash at the exchange rate of L. 100 per point and paid to you privately at the end of the experiment.

The four empty cells are numbered from 1 to 4 so that the software can recognize them. However, the specific position of the single squares in the cells is **irrelevant**. In other words, the four squares that you choose can be placed in the empty cells in any position you prefer. It is only important that they satisfy the pair of order relations assigned.

Further, we ask you to carefully read the single pairs of relations given. In this type of experiments it is easy to commit mistakes my simply misreading the data.

In order to begin the experiment, you will have to insert your identification number in the "number" window on your screen and then click OK. After this, the screen will display a set of squares on the left and some written text on the right. Before the actual experiment starts, you will go through a brief training session.

Please, we ask you to do the experiment in silence. Thank you.

Instructions for experiment 2

Today's experiment is divided in two parts. In the first part you will be asked to construct the representation of four interactive decision making situations, following a set of rules that will be explained to you shortly. This part of the experiment is strictly individual. Each of you will gain a fixed amount for any representation correctly constructed. In the second part of the experiment you will have to make a series of decisions, and your earnings will depend on your decisions and on the decisions of other participants. Your earnings in the second part of the experiment will be expressed in experimental points. One experimental point is worth $.75$ euros. At the end of the experiment your earnings in the first and second part will be summed up, converted in euros and paid to you privately in cash.

First part The experiment will regard interactive decision making situations called "games" (although the term is not to be intended in its everyday meaning in natural language: in our experiment a "game" is simply a decision situation in which earnings depend on the joint decisions of two players). You will be presented with four different "games", one at a time in sequence. All the games are two-player, and each player has always the choice between two moves, which are labelled with the two familiar symbols of *spades* and *clubs*. Therefore, for each game there are four possible combinations of moves of the two players: *spades-spades, spades-clubs, clubs-spades, clubs-clubs*. These four combinations generate four different scenarios labelled A, B, C and D, which differ in the payoffs that the they imply for the players. The payoffs in each game range from a minimum of 1 point to a maximum of 4 points, and are represented as squares of different size and different color. A darker square means a higher payoff for one player, while a bigger square means a higher payoff for the other player. Please, observe the screen in front of you: each of you can see a set of 16 squares of different size and color in the upper left portion of the screen. Each square represents a different pair of payoffs. In the lower left portion the four combinations of moves are displayed and, below each combination, the resulting scenario (A, B, C and D). At the bottom you can see four empty cells. For each game, you will be provided with the criteria specifying the relative ordering of your payoff and the other player's payoff in the four different scenarios. The criteria will appear to you in the text window in the right portion of the screen. You will have to select, among the 16 squares that are available to you, the 4 squares that according to you satisfy the ordering criteria given, and place each square in the corresponding cell. The criteria that you'll have to follow concern the relative ordering of the size and color of the squares, which correspond to the two players' payoffs.

For example, you will be asked to represent the payoffs of a game in which the squares associated with the different scenarios satisfy the following ordinal criteria regarding size and color:

1. COL: $A > B > C > D$
2. SIZE: $B > A > D > C$

The first condition states that the square of scenario A must be darker than the square of scenario B, which must be darker than the C square, and so forth (we establish by convention that the symbol "$>$" means "darker than"). The second condition states that the square of scenario B must be greater than the square of scenario A, etc. Remember that SIZE and COLOR of the squares represent the two players' payoffs.

Your task in each game is to select the 4 squares, among the 16 you have, that satisfy the two conditions simultaneously and place each square in the corresponding cell.

Technically, to place a square in a cell you must click with the mouse on the square and then click on the cell in which you want to place it, as we will show you now. To change your choice, you just have to click once on the new square and on the cell again. When you have finished, click with your mouse on the "choice" button.

There will be four different games which will appear to you in sequence. For each game correctly constructed, you will earn a payoff of 3 euros. For each pair of conditions, there is only one correct solution.

The four games are the same for all of you, but in the first part of the experiment they will appear to each of you in a different order, determined randomly by the computer program.

IMPORTANT: you will have a time limit of 120 seconds to complete each task (the passing of time will be visualized by a bar at the bottom of the screen). If you haven't completed a representation when the two minutes have elapsed, the task will be completed arbitrarily by the computer program, and your payoff for the task will be zero. In order to register your choice, remember to click on "choice" when you are done.

Second part In the second part of the experiment you will have to play the four games in sequence; i.e., for each game that you have represented in the first part, you will have to choose a move among the two available, *spades* and *clubs*. The screen will display each game exactly as you represented it in the first part. In each game, the computer program will connect you with another participant chosen randomly, and your earnings will be determined by the combination of yours and the other's choices. At the beginning of each game, you will also be assigned randomly to the variable associated with your payoff (COLOR or SIZE of the squares). Therefore, in some of the games, your payoff may be associated with the squares' size, in other games with their color. Clearly, if you have been assigned to COLOR in a game, the other player you are paired with has been assigned to SIZE and vice-versa. By convention, we establish that the COLOR player will always be the first player of the couple, whereas the SIZE player will be the second. The pairings between participants will be determined randomly and will be presumably different in each game. For each game, you will see on your screen the representation that you have constructed in the first part. Please, have a look at the screen: on top of the game there will be the indication of your payoff variable for that game (SIZE or COLOR); below that you will see the four possible combinations of moves, and below that your two possible moves: the bottom part of the screen will report the squares representing your and your opponent's payoffs in each scenario. To make the reading of the game easier, your moves are circled in red so that you can distinguish them from your opponent's moves. You have to be aware that your payoffs will be calculated as a function of both players' choices *on the basis of the correct game representation*. We remind you that for any pair of criteria there is only one correct representation. You can make your choice by simply clicking on the corresponding symbol. You will not be allowed to know the other player's choices or your payoffs until the whole experiment is over.

WARNINGS:

For technical reasons, all participants in the experiment must proceed synchronized. Therefore, if you finish a task before the others, please wait in silence that all have completed theirs. When you have completed a task and have clicked on the *choice* button, please pay attention to the lower left corner of the screen: you will read either "wait" or "new round: make your choices". If you have questions, please raise your hand and somebody will come to you. Finally, you are not allowed to use paper and pencil. Are there any questions?

	x	y
x	3,3	2,2
y	1,1	4,4

Table 1: A coordination game

	x	y
x	1,4	3,2
y	4,1	2,3

Table 2: A game of conflict

	x	y
x	1,1	4,2
y	2,4	3,3

Table 3: A game of chicken

	x	y
x	2,2	4,1
y	1,4	3,3

Table 4: A prisoner's dilemma

	group 1 (N=14)			group 2 (N=16)			Tot. N=30
Bi-order	sequence	number	freq.	sequence	number	freq.	freq.
Isot. proj.	2	13	.93	3	15	.94	.93
Antit. proj.	1	9	.64	4	12	.75	.70
Proj.	4	7	.50	2	10	.62	.57
Non Proj.	3	6	.43	1	6	.37	.40
	group 1 (N=20)			group 2 (N=20)			Tot. N=40
Isot. proj.	2	19	.95	3	19	.95	.95
Antit. Proj.	1	15	.75	4	18	.90	.82
Proj.	4	15	.75	2	14	.70	.72
Non Proj.	3	10	.50	1	9	.45	.47

Table 5: Experiment 1: numbers and relative frequencies of correct answers in the four tasks in the Venice pool (upper part) and in the Trento pool (lower part). The second column reports the order with which the tasks were presented to subjects in the two different sub-groups.

task	correct answers
isotonic	66 (.94)
antitonic	54 (.77)
projective	46 (.66)
non-projective	31 (.44)

Table 6: Experiment 1: numbers and relative frequencies of correct answers pooled across sessions.

Correlation coefficient		p-value (one-tailed)
Kendall's tau	$.310$	$.05$
Spearman's rho	$.362$	$.05$

Table 7: Experiment 1: correlation coefficients between individual score in the Wechsler digit span test and the number of correct responses in the experiment.

task	unsuccessful subjects	successful subjects	p-value
coordination (isotonic)	5.3 (n=3)	5.9 (n=35)	.563
conflict (antitonic)	5.2 (n=5)	6 (n=33)	.011
chicken (projective)	5.4 (n=9)	6.03 (n=29)	.033
PD (non-projective)	5.8 (n=19)	6 (n=19)	.516

Table 8: Average STM score of subjects who err and of subjects who don't err in each task. The last column reports the p-value of the T-test for equality of means.

task	correct answers
coordination (isotonic)	59 (.87)
conflict (antitonic)	53 (.78)
chicken (projective)	45 (.66)
PD (non-projective)	35 (.51)

Table 9: Experiment 2: numbers and relative frequencies of correct answers in the representation task

	isotone	antitone	projective	non-projective
chicken game (n=17)	6	0	9	2
PD (n=23)	5	9	3	6

Table 10: Experiment 2: empirical distribution of erroneous representations classified by typology of bi-order.

		games			
		Coordination	Conflict	Chicken game	PD
row	x	0.45	0.19	0.36	0.68
	y	0.55	0.81	0.64	0.32
col	x	0.16	0.48	n.a.	n.a.
	y	0.84	0.52	n.a.	n.a.

Table 11: Experiment 2: relative frequencies of choices of actions in each game, disaggregated by row and column role for the non-symmetric games.

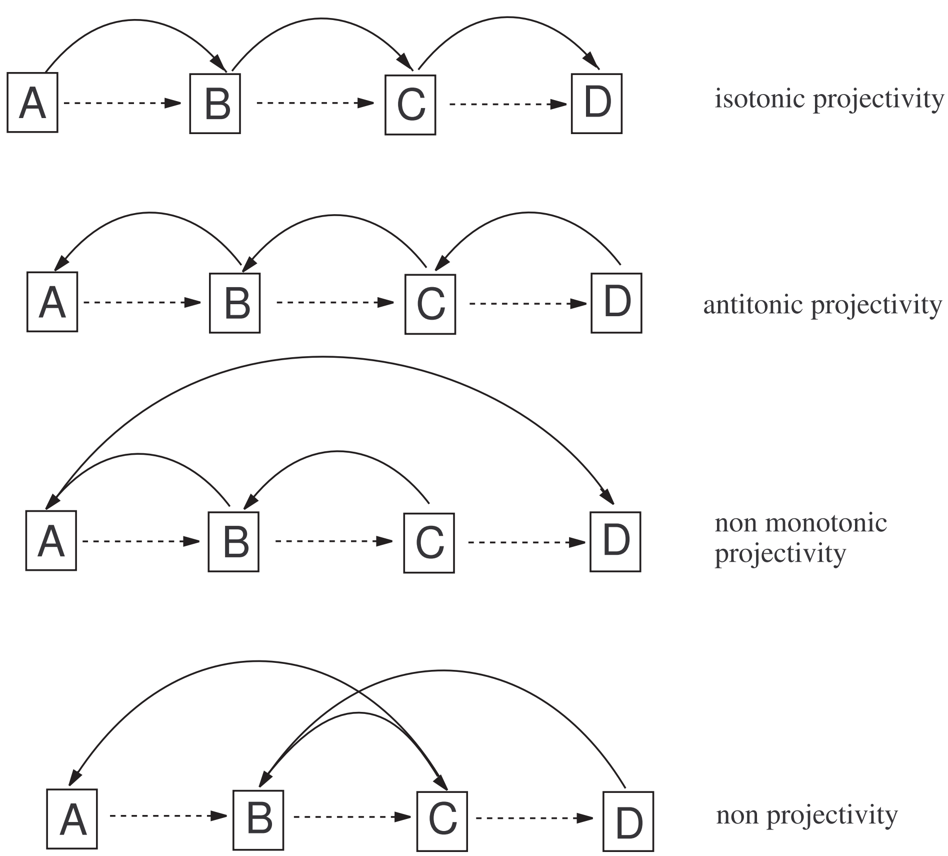

Figure 1: Four examples of bi-order structures.

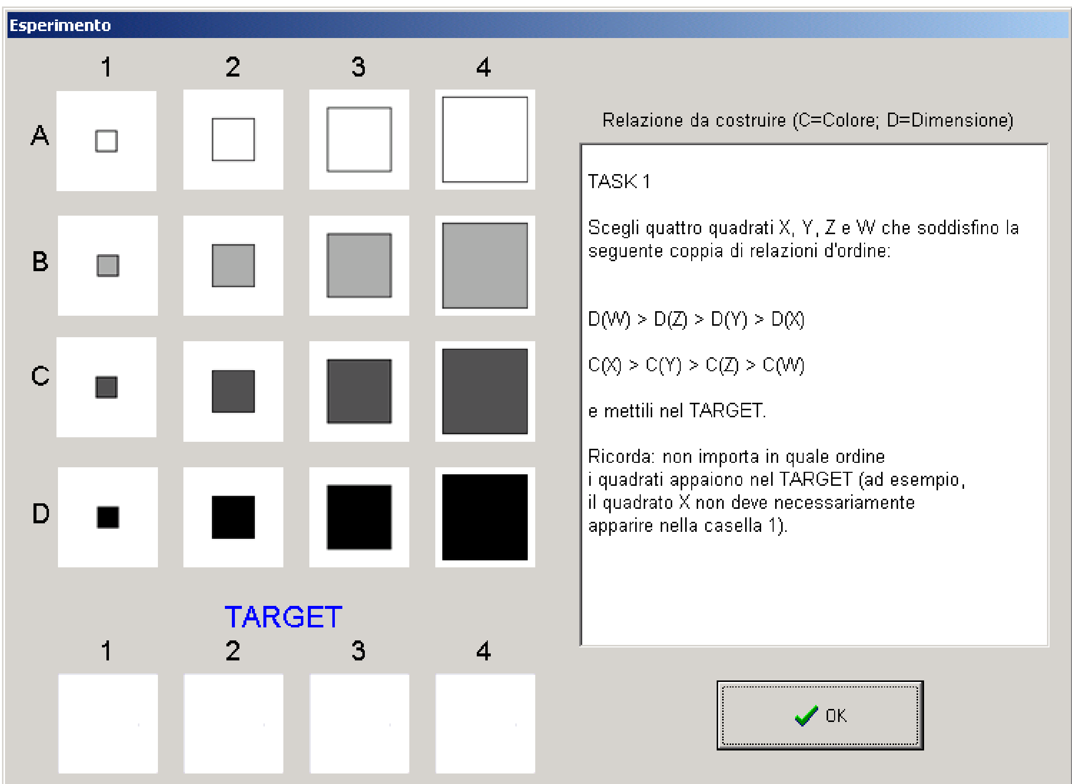
Esperimento
1
2
3
4
A
B
C
D
Relazione da costruire (C=Colore; D=Dimensione)
TASK 1
Scegli quattro quadrati X, Y, Z e W che soddisfino la seguente coppia di relazioni d'ordine:
D(W) > D(Z) > D(Y) > D(X)
C(X) > C(Y) > C(Z) > C(W)
e mettili nel TARGET.
Ricorda: non importa in quale ordine i quadrati appaiono nel TARGET (ad esempio, il quadrato X non deve necessariamente apparire nella casella 1).
TARGET
1
2
3
4
OK

Figure 2:

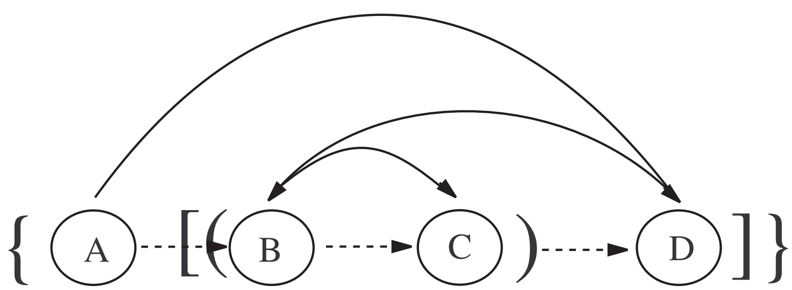

Figure 3:

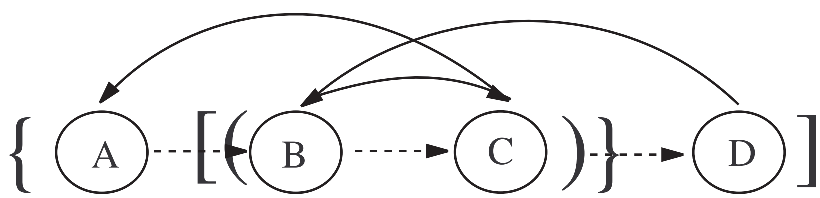

Figure 4:

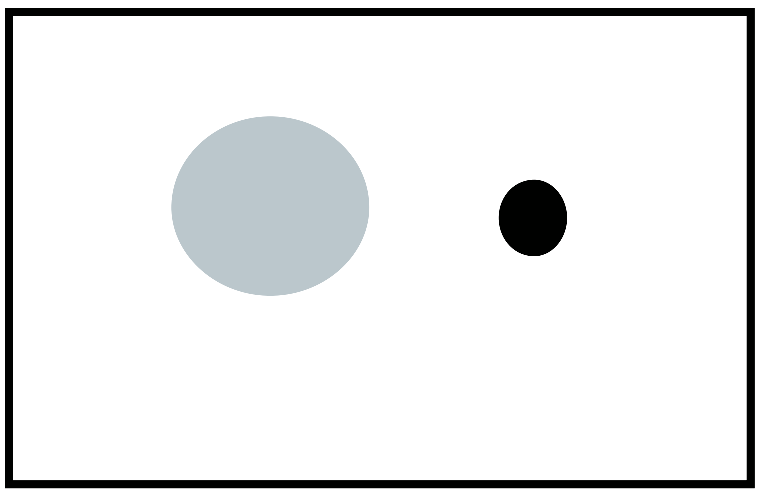

Figure 5:

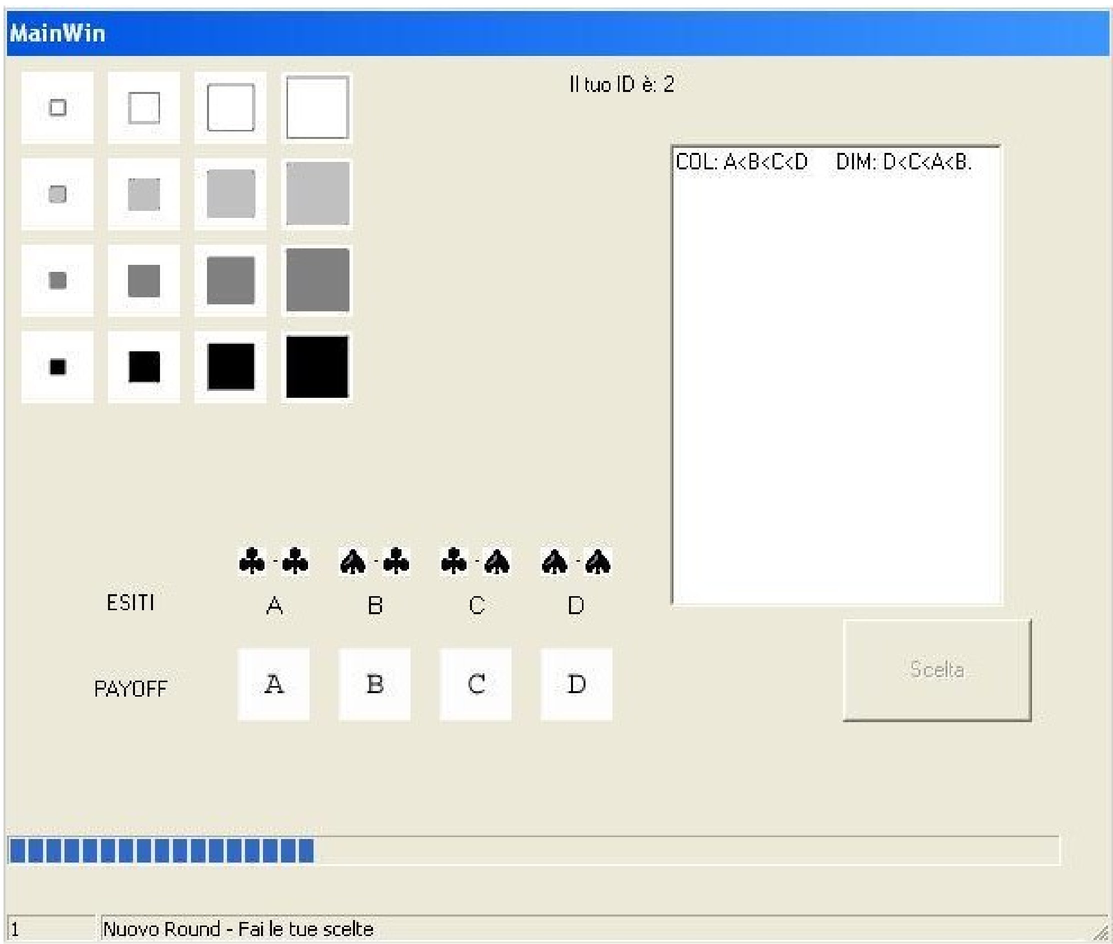
MainWin
Il tuo ID è: 2
COL: A<B<C<D DIM: D<C<A<B.
ESITI
A
B
C
D
PAYOFF
A
B
C
D
Scelta
1
Nuovo Round - Fai le tue scelte

Figure 6:

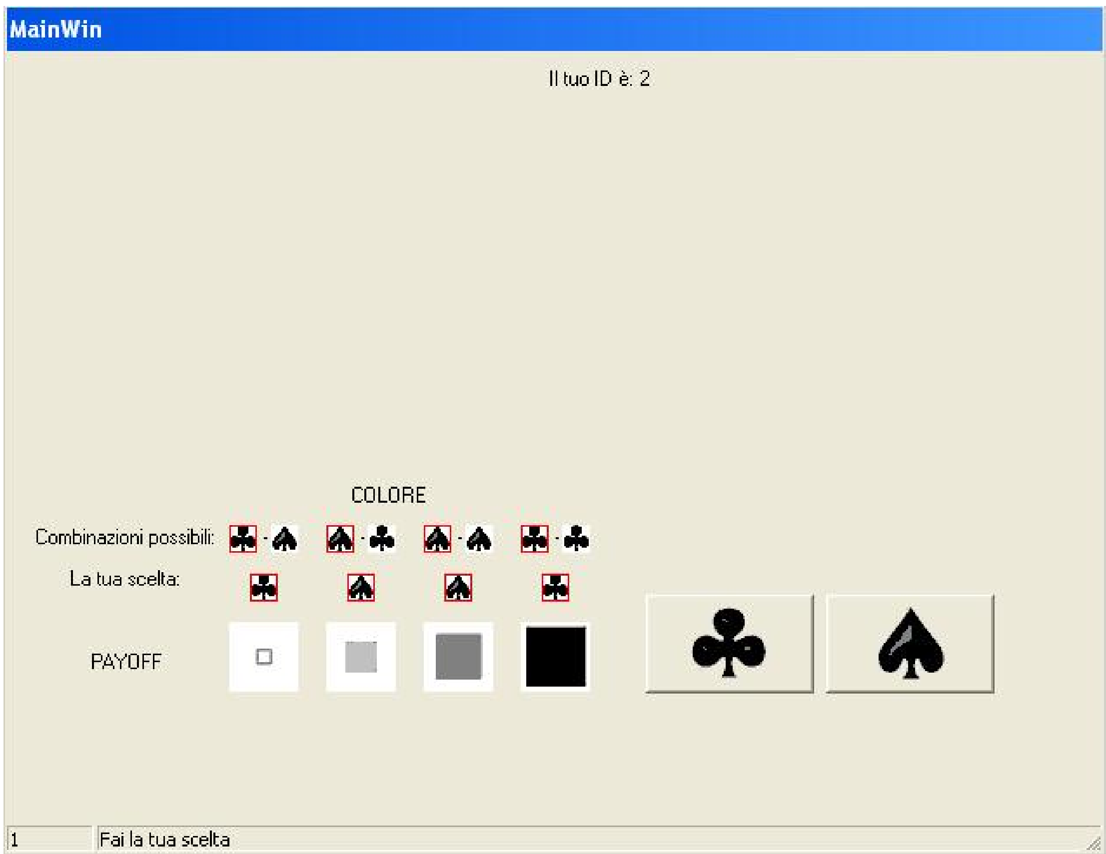
MainWin
Il tuo ID è: 2
COLORE
Combinazioni possibili:
La tua scelta:
PAYOFF
1
Fai la tua scelta

Figure 7: