Dynamic inconsistency of non-expected utility agents in an Allais-type experiment

The experiment intends to provide a test of the dynamic inconsistency of agents whose preferences violate expected utility theory through violation of the independence axiom; and of the strategies available to the agents in this decision context.

In a context of choice under risk or uncertainty, dynamically inconsistent choices occur when the preference orderings over risky (uncertain) outcomes are non linear in the probabilities, that is, violate expected utility theory through violation of the independence axiom. When applied to sequential decisions, non-linearity may cause optimal strategies to be dynamically inconsistent (Hammond (1988a,b;1989); Machina, 1989; McClennen, 1990).

Two aspects of the debate on the general problem of dynamic inconsistency are particularly relevant:

1) the investigation on the principles of dynamic choice (McClennen, 1990, Cubitt, 1996, Segal, 1990). Not many contributions have investigated this aspect through experiments (Cubitt, Starmer, Sugden 1998, Paradiso and Hey, 1999).

2) the investigation on the different strategies of choice available to the agent in a decision situation with a potential for dynamic inconsistency (McClennen, 1990, Machina, 1989): Machina develops a choice strategy (MNEU) which allows to avoid dynamic inconsistency, by removing Hammond's assumption of consequentialism. If this assumption holds, at any decision node the agent ignores any part of the tree which cannot be reached by moving forward from that node. But if the risk borne in the past can be considered relevant to current decisions in a way consistent with the agent's original preferences, it is possible to formalise a choice strategy where a non-expected utility agent is consistent through time. This model of choice is substantially equivalent, even if differently formalised, to McClennen's model of Resolute Choice (RC). The non-expected utility agent (NEU) who is consequentialist, will be dynamically inconsistent (and myopic: Strotz 1956, Hammond 1976). Two other kinds of agent can be considered in this choice context: the expected utility (EU) agent, who is dynamically consistent, and·the Sophisticated Chooser (SC) (Machina, McClennen).

The experiment tests the potential inconsistency of agents with non-expected utility preferences in the Allais-type dynamic choice situation analysed by Machina (1989) - where there is a potentiality of inconsistency for non-expected utility subjects - considering first whether agents behave according to expected utility theory in a static context; and then - in case they do not - which model of

behaviour in dynamic situations they adopt in case inconsistency is avoided - sophisticated choice or Machina- non-expected utility/Resolute choice.

>From the results of the experiment two different elements emerge:

1. a high number of expected utility choices in the static decision problem are consistent with expected utility, and correspond to the pair of options which are more risky and have the highest expected payoff. This result has been interpreted as a consequence of the change in size of the payoffs with respect to the original hypothetical ones, as supported by a study by Conlisk (1989) and by possible formal explanations;

2. 30% of the subjects shift strategy and/or preference pattern from the static to the pre-committed dynamic problems, 25% of which maintain the new strategy also at the choice node. All the subjects who changed strategy were EU in the static and some type of NEU in the dynamic problems. The interpretation given shows that the shifts in strategy (all from EU to NEU) and/or preferences pattern of the subjects from the static to the dynamic problems imply violation of the reduction axiom. It emerges that violation of reduction helps also to define sophisticated choice: all shifts in strategy from non-expected utility to expected utility are consistent with sophisticated choice.

Two considerations emerge from the results:

1. nearly all the choices in the static decision problem are consistent with expected utility, and are for the two risky prospects. This is likely to be an effect of the probability/payoff structure, which makes the risky outcomes very attractive, as will be discussed below;
2. there is a shift in choices from the static to the pre-committed dynamic problems, with an increase of choices consistent with NEU strategies.

In other words, it is possible to outline two different kinds of inconsistency.

(i) Some subjects are inconsistent with the strategy adopted in the static problem when choosing in the dynamic problems. For example, they choose according to EU in the static and to NEU in the dynamic problems;

(ii) some subjects are consistent with the static choice strategy, but change the pattern of preference, for example, are always consistent with EU, but choose the risky pair of options in the static, and then shift their choice to the safe pair in the dynamic problem. The subjects who are inconsistent in their strategy also change their preference pattern through problems.

2.1 The experiment

In the experiment, the subjects played the following four decision problems in sequence:

Problem 1

You have to choose one of the following two options:

Option A: No ticket will be drawn from the bag. You will receive £5.

Option B: The experimenter will draw a ticket from the bag. If the ticket has a number between 1 and 20, you will receive £10. If it has a number between 21 and 95 you will receive £5. If it has a number between 96 and 100, you will receive nothing.

Now choose one of the following two options:

Option C: The experimenter will draw a ticket from the bag. If the ticket has a number between 1 and 20, you will receive £10. If it has a number between 21 and 100 you will receive nothing.

Option D: The experimenter will draw a ticket from the bag. If the ticket has a number between 1 and 25, you will receive £5. If it has a number between 26 and 100 you will receive nothing.

Problem 2

You have to choose one of the following two options:

Option A: No ticket will be drawn from the bag. You will receive £5.

Option B: The experimenter will draw a ticket from the bag. If the ticket has a number between 1 and 80, you will receive £10. If it has a number between 81 and 100, you will receive nothing.

Problem 3

The experimenter will draw a ticket from the bag. If the ticket has a number between 1 and 75, you will receive £5. If it has a number between 76 and 100, you have to decide which of the following two options you want to receive:

Option A: No ticket will be drawn. You will receive £5.

Option B: The experimenter will draw a second ticket from the bag. If the ticket has a number between 1 and 80, you will receive £10. If it has a number between 81 and 100 you will receive nothing.

Problem 4

The experimenter will draw a ticket from the bag. If the ticket has a number between 1 and 75, you will receive nothing. If it has a number between 76 and 100, you have to decide which of the following two options you want to receive:

Option A: No ticket will be drawn. You will receive £5.

Option B: The experimenter will draw a second ticket from the bag. If the ticket has a number between 1 and 80, you will receive £10. If it has a number

between 81 and 100 you will receive nothing.

Problem 1 is a static choice problem and is one version of the Allais paradox problem considered in section 1.2.1 of Chapter 1, where options A, B, C and D (holding the reduction of compound lotteries axiom) are the prospects a_1=(1 chance of 5£), a_2=(.20 chance of 10£; .75 chance of 5£; .05 chance of 0£), a_3=(.20 chance of 10£; .80 chance of 0£) and a_4=(.25 chance of 5£; .75 chance of 0£) respectively.

Problem 2 represents the direct choice between the lottery B = (10£,.80; 0,.20) and the sure outcome 5£, equivalent to the one considered in section 1.3.1 of the previous chapter.

Problems 3 and 4 are the dynamic choice decision problems represented by the left and right hand-side decision trees in section 1.3.1, Figure 1.4, with the following outcomes and probabilities:

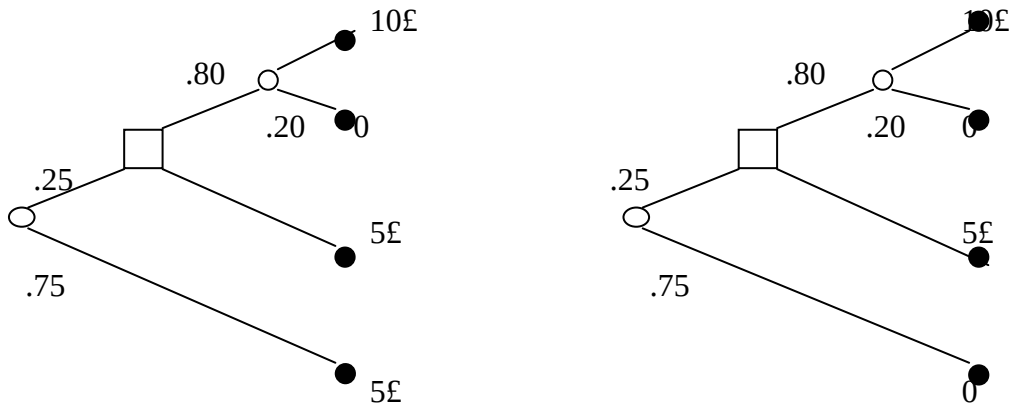

Figure 2.1

All the considerations made above about an agent's behaviour hold under these different outcomes and probabilities. The outcomes in the original Allais example had to be changed, given that the lotteries in the experiment were played out for

real. As for the probabilities, the probabilities used here allowed to use an urn of 100 tickets to determine the lottery probabilities for all problems and all problem stages.

In the experiment the problems were played by the subjects in the following way.

All subjects took part in the experiment in two different days. In the first day, all subjects were divided in two groups. They were presented the four decision problems on four different decision sheets, in sequence. All decisions had to be taken before any chance was played out. In the following two days, subjects were asked to come back in groups of four for each session. Their decisions on the four problems were played out at this stage. In Problems 3 and 4, in case the decision node were reached (and hypothetically otherwise), the subjects were given the chance to reconsider the decision they had pre-committed to the previous day.

Thus, all the subjects knew only at this stage that the pre-commitment on the decisions taken on the previous session of the previous day was not binding, which they did not know when actually taking those decisions. All decisions were taken by all players before any chance were played out, and the opportunity of changing mind were offered to any subject. Differently, knowing that pre-commitment was not binding for the players in a previous session could have influenced the behaviour of the subjects playing in later sessions.

At the end of the second session, one out of the four problems was chosen at random, and the subjects were paid according to the outcome obtained in that problem, plus their participation fee.

Consider now how the choices in the decision problems above can help in predicting the different choice strategies adopted by the agent (or, alternatively, identifying the different kinds of agents considered), and therefore testing for their dynamically inconsistent behaviour.

Problem 1 allows to tell apart expected utility from non-expected utility agents. Problems 2, 3 and 4 allow to find out which kind of non-expected utility agent the

subject is, whether NEU, MNEU or SC. In order to show this, consider how the choices of the four subjects would differ in the four problems.

2.1.1 The predicted choice strategies of the different kinds of agent

Consider first the NEU non-expected utility agent and the MNEU Machina-non-expected utility agent. Both have exhibited a preference pattern which violates independence in Problem 1. Suppose this is $a_1 \succ a_2$ and $a_3 \succ a_4$.

Problem 2. The subject faces the choice between lottery B and 5£ for sure. Suppose he chooses lottery B over 5£.

Problem 3 (Left hand-side tree).

(i) pre-committed choice. The subject is asked what he would choose before chance is played out.

A NEU will choose option A/Down ($a_1 \succ a_2$)

A MNEU will choose A/Down ($a_1 \succ a_2$)

(ii) at choice node. Chance is played out. With 25% of chance the subject ends up at the decision node.

Problem 4 (Right hand-side tree).

(i) pre-committed choice. The agent is asked what he would choose before chance is played out.

NEU chooses B/Up ($a_3 \succ a_4$)

MNEU chooses B/Up ($a_3 \succ a_4$)

(ii) at choice node. Chance is played out. With 25% chance the subject ends up at the decision node.

NEU chooses B/Up for lottery B

A NEU chooses B/Up (B ≻ 5£), inconsistently with the decision he pre-committed to at the beginning of the tree.

A MNEU chooses A/Down (a_1), consistently with his choice at the beginning of the tree. Both at the initial node, before uncertainty is resolved, and in the middle of the tree, at the decision node, the MNEU agent always faces a_1 and a_2, not B and 5£.

MNEU chooses B/Up for a_3. No subject is inconsistent.

Problem 2. The subject faces the choice between lottery B and 5£ for sure. Suppose he chooses 5£ over lottery B.

Problem 3 (Left hand-side tree).

(i) pre-committed choice. The subject is asked what he would choose before chance is played out.

A NEU will choose A/Down ($a_1 \succ a_2$)

A MNEU will choose A/Down ($a_1 \succ a_2$)

(ii) at choice node. Chance is played out. With 25% of chance the subject ends up at the decision node.

A NEU chooses A/Down (5£ $\succ$B)

A MNEU will choose A/Down for a_1. No subject is inconsistent.

Problem 4 (Right hand-side tree).

(i) pre-committed choice. The agent is asked what he would choose before chance is played out.

NEU chooses B/Up ($a_3 \succ a_4$)

MNEU chooses B/Up ($a_3 \succ a_4$)

(ii) at choice node. Chance is played out. With 25% chance the subject ends up at the decision node.

NEU chooses A/Down for 5£, being inconsistent with previous choice.

MNEU chooses B/Up for a_3, avoiding dynamic inconsistency.

Consider now the choice of an agent with Expected utility EU preferences.

Expected utility preferences do not violate replacement separability. In Problem 1, they rank $a_1 \succ a_2$ and $a_4 \succ a_3$; or $a_2 \succ a_1$ and $a_3 \succ a_4$.

Problem 2. Suppose the subject chooses lottery B over 5£

Problem 3 (Left hand-side tree)

(i) pre-committed choice. EU chooses B/Up, $a_2 \succ a_1$

(ii) at choice node. He chooses B/Up at decision node

Problem 4 (Right hand-side tree).

(i) pre-committed choice. EU chooses B/Up, $a_3 \succ a_4$

(ii) at choice node. EU chooses B/Up at decision node

Problem 2. Suppose the subject chooses 5£ over lottery B.

Problem 3 (Left hand-side tree).

(i) pre-committed choice. EU chooses A/Down, for a_1

(ii) at choice node. He chooses A/Down, for 5£

Problem 4 (Right hand-side tree).

(i) pre-committed choice. EU chooses A/Down, for a_4

(ii) at choice node. He chooses A/Down, for 5£

Then, the EU subject who prefers the lottery to 5£ for sure, in Problem 3 (left hand-side tree) chooses differently from both NEU and MNEU. In Problem 4 (right hand-side tree) EU chooses like NEU and MNEU. In fact, all are consistent here.

The EU who prefers 5£ for sure over lottery B, in the left hand-side tree chooses like NEU and MNEU (all are consistent). In the right hand-side tree, he chooses differently.

Consider now the choice strategy of sophisticated choice SC. In problem 1 SC exhibits non-expected utility preferences. In all the dynamic decision problems, SC is always dynamically consistent. The SC who prefers lottery B over 5£, chooses always Up. The SC who prefers 5£ over lottery B chooses always Down. As discussed in the previous chapter, the SC agent will behave like an EU agent in the dynamic problems, but choose as a NEU agent in the static problem.

The need to distinguish SC from EU agents when they choose in a dynamic problem is the reason why, before the dynamic version of the Allais paradox was played, choices in a static Allais paradox are being tested. These allowed to distinguish EU from NEU subjects, and then EU agents from sophisticated

choosers who are non-expected utility, but dynamically consistent. Consistent NEU may behave according to either MNEU or SC.

One more thing to consider is the possibility that the agent exhibits the preference pattern which is considered uncommon, but can occur (and in fact occurred) in the experiment, that is, $a_2 \succ a_1$ and $a_4 \succ a_3$ for NEU or $a_3 \succ a_4$ for EU.

As seen above, if an EU has preferences for $a_2 \succ a_1$ and $a_3 \succ a_4$, he will choose lottery B over 5£ and always Up.

If he prefers $a_1 \succ a_2$ and $a_4 \succ a_3$, he will choose 5£ over the lottery B, and always Down.

Consider the choice of NEU and MNEU.

Problem 2. Subject chooses lottery B over 5£ for sure.

Problem 3. (Left hand-side tree)

(i) pre-committed choice.

A NEU chooses B/Up (a_2)

A MNEU chooses B/Up (a_2).

(ii) at choice node. Chance is played out.

A NEU chooses B/Up (lottery)

A MNEU chooses B/Up (a_2). No subject is inconsistent.

Problem 4.(Right hand-side tree).

(i) pre-committed choice

NEU chooses A/Down (a_4).

MNEU chooses A/Down (a_4)

(ii) at choice node.

NEU chooses B/Up for lottery B, being inconsistent with previous choice.

MNEU sticks to A/Down, for a_4

Problem 2 Subject chooses 5£ over lottery B.

Problem 3 (Left hand-side tree)

A NEU chooses B/Up (a_2)

A MNEU chooses B/Up (a_2).

(ii) at choice node.

A NEU chooses A/Down (5£), being inconsistent with previous choice.

A MNEU chooses B/Up (a_2).

Problem 4. (Right hand-side tree)

(i) pre-committed choice

NEU chooses A/Down (a_4).

MNEU chooses A/Down (a_4)

(ii) at choice node.

NEU chooses A/Down for 5£.

MNEU chooses A/Down, for a_4.

The different choices taken by the four kind of agents in the different problems are summarised in the Table below. The shaded choices indicate the cases of inconsistency for the NEU agent.

	EU		NEU		MNEU		SC	
	$a_1 \succ a_2$ $a_4 \succ a_3$	$a_2 \succ a_1$ $a_3 \succ a_4$	$a_1 \succ a_2$ $a_3 \succ a_4$	$a_2 \succ a_1$ $a_4 \succ a_3$	$a_1 \succ a_2$ $a_3 \succ a_4$	$a_2 \succ a_1$ $a_4 \succ a_3$	$a_1 \succ a_2$ $a_3 \succ a_4$	$a_2 \succ a_1$ $a_4 \succ a_3$
Problem 1	A - D	B - C	A - C	B - D	A - C	B - D	A - C	B - D
Problem 2	A (£5 for sure)	lottery B	A (£5)	B	A (£5)	B	A (£5)	B	A (£5)	B	A (£5)	B
Problem 3	A/ Down	B/ Up	A (a_1)	A (a_1)	B (a_2)	B (a_2)	A (a_1)	A (a_1)	B (a_2)	B (a_2)	A (a_1)	B
Problem 3 at choice node	------	-----	A (£5)	B	A (£5)	B	A (a_1)	A (a_1)	B (a_2)	B (a_2)	A (£5)	B
Problem 4	A	B	B (a_3)	B (a_3)	A (a_4)	A (a_4)	B (a_3)	B (a_3)	A (a_4)	A (a_4)	A	B
Problem 4 at choice node	------	------	A (£5)	B	A (£5)	B	B (a_3)	B (a_3)	A (a_4)	A (a_4)	A (£5)	B

2.2 The results

The subjects who took part in the experiment were 40 undergraduate and graduate students of different disciplines at the University of York.

Let us consider first the choices of the subjects in the static problem 1, and the pre-committed choices in the dynamic problems 3 and 4. This will allow to distinguish EU from NEU agents. It is expected that a subject will choose the same prospects in both the static and the dynamic problems. Later, the choices of the subjects at the decision nodes of problems 3 and 4 will be considered to check the consistency of the subjects with their pre-committed choices, thus distinguishing the different types of NEU agents.

2.2.1 Static and pre-committed dynamic choices

Consider the pre-committed choices of the subjects in the different problems, the static problem 1 and the dynamic problems 3 and 4.

The choices highlighted in the diagonal correspond to choices consistent with expected utility. The other choices are non-expected utility choices.

Static choices - Problem 1. Number of choices out of 40.

	Sure outcome: D (a_4)	Risky outcome: C (a_3)
Sure outcome:A (a_1)	0	2
Risky outcome:B (a_2)	2	36

Dynamic pre-committed choices - Problems 3 and 4

Problem 4 / Problem 3	Sure outcome:A (a_4)	Risky outcome:B (a_3)
Sure outcome:A (a_1)	3	1
Risky outcome:B (a_2)	6	27*

*3 SC choices are in this box. They coincide with EU choices, but cannot be considered EU.

27 subjects (26 EU and 1 NEU) are consistent with both strategy and preference pattern;

12 subjects are inconsistent with their static choice strategy and/or preference pattern. Out of these, though, 10 subjects behave according to a well defined strategy in the dynamic choice problems, both pre-committed and at decision node; 2 subjects violate the adopted strategy at the decision node;

1 subject is not consistent with any strategy.

Details of the choices in the different sessions and problems are given for all subjects in the table at the end of the chapter.

Consider first the EU subjects.

Out of the 36 static EU choices in Problem 1, 26 remain consistent with EU in the dynamic problems 3 and 4, all exhibiting always the pattern a_2 - a_3.

What happens to the other EU choices? 7 EU subjects behave as NEU in the dynamic problems; 3 behave according to EU but change their preference pattern. Consider their choices in more detail:

5 EU a_2 - a_3 behave as MNEU a_2 – a_4 in dynamic problems (inconsistent in strategy and preferences)

1 EU a_2 - a_3 behaves as MNEU choosing a_1 - a_3 (inconsistent in strategy and preferences) (session 5)

1 EU a_2 - a_3 behaves as NEU a_2 – a_4 (inconsistent in strategy and preferences), and dynamically inconsistent in problem 4, where he changes for the lottery option B at the decision node, choosing at the node as in problem 2.

1 EU a_2 - a_3 chooses as EU a_1 - a_4, (inconsistent in preferences)

1 EU a_2 - a_3 chooses as EU a_1 - a_4, but is dynamically inconsistent in problem 4 (changes to a_3, and then is inconsistent in strategy and preferences)

1 EU a_2 - a_3 chooses as EU a_1 - a_4 but is not consistent in all problems with any strategy, as she chooses option B in Problem 2.

Consider the non-expected utility NEU subjects.

All 4 subjects who are NEU in problem 1, are NEU in problems 3 and 4.

1 NEU a_2 - a_4 behaves as SC a_2 - a_3

1 NEU a_1 - a_3 behaves as SC a_2 - a_3

1 NEU a_2 - a_4 chooses according to SC a_2 - a_3, but is dynamically inconsistent in problem 4, therefore not consistent with SC.

1 NEU $a_1 - a_3$ behaves as MNEU a_2 - a_4, and is inconsistent in the preference pattern.

NEU subjects remain NEU in dynamic problems, and 3 out of 4 behave according to the SC strategy (1 SC subject is however not consistent with the strategy, as he 'changes mind' at the decision node).

2.2.2 Choices at decision nodes

Consider now the dynamic inconsistency of the NEU subjects at the decision nodes in problems 3 and 4.

As seen above, 26 subjects are consistent with EU, therefore dynamically consistent; 3 are EU in both static and dynamic problems, despite they change preference pattern, and are therefore dynamically consistent; 1 does not follow any strategy and is therefore irrelevant to the problem of dynamic inconsistency.

Out of the other 10 subjects, 4 subjects are NEU in static problem 1 and dynamic problems 3 and 4; 6 subjects are EU in the static, but NEU in the dynamic problems and can therefore be tested for dynamic inconsistency.

It results that only 1 subject wanted to change his mind at the decision node, therefore identifying with a straight NEU subject. This subject had chosen the lottery over the sure outcome in problem 2, and prospects a_2 - a_4 in the dynamic pre-committed problems, and reconsidered his choice to a_3 at the decision node in problem 4, as predicted by NEU in that problem.

2 of the subjects who always behaved as NEU in all problems, behaved according to SC, therefore consistently with the pre-committed choice: one subject chose prospects $a_1 - a_3$ in problem 1, the lottery in problem 2 and prospects $a_2 - a_3$ in problems 3 and 4, therefore changing pattern, but behaving according to the SC strategy; the second subject chose prospects a_2 - a_4 in problem 1, $a_2 - a_3$ in problems 3 and 4, and the lottery in problem 2, behaving according to

SC. A third subject behaved according to SC in the pre-committed problems, but was dynamically inconsistent in problem 4, therefore violating SC.

6 subjects behaved according to MNEU.

Out of these, 1 subject was NEU in all problems, choosing prospects $a_1 - a_3$ in problem 1, and changing preference pattern to $a_2 - a_4$ in the dynamic problems, choosing the sure option A in problem 2 and prospects $a_2 - a_4$ in problems 3 and 4. Inconsistency could occur for this subject at the decision node of problem 3, where he faced the same options of problem 2. As predicted by MNEU, the subject did not reconsider his decision here, treating choice at decision node differently from choice as independent of previous history.

5 subjects were EU in the static problem 1, with preferences for $a_2 - a_3$, and NEU in problems 3 and 4. Choices of these subjects are different, though all consistent with MNEU. 3 subjects chose the lottery option B in problem 2, and options $a_2 - a_4$ (B – A) in problems 3 and 4. Inconsistency could occur in problem 4 at choice node, but all subjects were consistent. 1 subject chose the sure option A in problem 2, and $a_2 - a_4$ in problems 3 and 4. Inconsistency which was possible at the decision node of problem 3 did not occur. The last subject chose the lottery in problem 2 and $a_1 - a_3$ (A – B) in problems 3 and 4. Inconsistency could occur here at the choice node of problem 3.

Given the high number of EU choices, the number of subjects whose inconsistency at the choice node of the dynamic problems can be tested is limited. However, out of the 10 subjects only one can be identified with a straight dynamically inconsistent NEU subject. 6 subjects behave according to the predictions of a Machina MNEU, and 2 according to SC (a third SC is inconsistent at choice node). This seems to contradict the argument that subjects who exhibit non-expected utility preferences end up being inconsistent when choosing in even the simplest dynamic choice situation.

There are two elements which emerge from the results that require an explanation. One is the high number of expected utility choices (90%) in the static

problem, which could be a consequence of the payoff structure of the choice problems. The other is the shift in strategy choice and/or preference pattern of 30% of the subjects from the static to the pre-committed dynamic problems, 25% of which are consistent with the new strategy also at the choice node. All the subjects who changed strategy were EU in the static and some type of NEU in the dynamic problems.

2.2.3 The effect of real payoffs

As noted above, in the static choice problem 1, where the Allais prospects are offered to subjects, nearly all decisions were for the expected utility pair of options $a_2 - a_3$.. This result can be interpreted as a consequence of the change in size of the payoffs. This consideration is supported by a study by Conlisk (1989), where the robustness of Allais behaviour is tested experimentally by introducing different variants of the original example.

In part of Conlisk's study, original payoffs were scaled down and options were played out for real. He finds that violations of expected utility represented by the Allais type behaviour were reduced when choices in the Allais decision problem concerned real small rather than hypothetical large payoffs. In Conlisk's pilot study real payoffs of size ($0, $5, $25) were used instead of the original hypothetical payoffs. These payoffs are similar in size to the ones used in the experiment described in this chapter[1].. It turned out that non-expected utility behaviour almost disappeared, as nearly all the subjects chose the pair of prospects corresponding to the pair $a_2 - a_3$ in the experiment. That is, nearly all subjects in both couples of options chose the more risky option with the highest expected payoff.

The conclusions to be drawn from this on the factors which cause the reduction of violations of independence are however not so clear. There are in fact two hypotheses to explain this, which are confounded: the reduction of violations can be due to the switch of the payoffs from large to small, and the reduction of violations can be due to the switch of the payoffs from hypothetical to real.

[1] Details of the design used are in Appendix IV of Conlisk (1989).

According to Conlisk's reasoning, if the first cannot be ruled out, the second cannot be accepted or rejected, and no conclusion can be drawn on the effect on Allais behaviour of real large payoffs from the small real payoffs case, as it is not possible to distinguish the two hypotheses with the data available.

Conlisk offers a very plausible reasoning in support of the first hypothesis, which he claims is also supported by the theory. When payoffs have been reduced to small amounts, it is not convenient for the subject to give up a higher expected value of option a_2 to eliminate the chance of getting nothing by choosing the sure option a_1. As for option a_3, it is always true that it yields a higher expected value by increasing only by little the chance of getting nothing with respect to a_4. Following this reasoning, the subject will choose the pair $a_2 - a_3$ instead of $a_1 - a_3$, therefore eliminating Allais behaviour. That means, when payoffs are large the concern for risk dominates the concern for expected payoff, when payoffs are small, the opposite occurs. According to Conlisk, the dominance of expected payoffs on risk when payoffs get small is formally explained both in the context of expected utility theory and in the context of Machina's fanning-out model of generalised expected utility theory[2].

It is to be noted however that violations are also reduced when small payoffs are hypothetical rather than real. The same result occurs in the pilot study of the experiment described in this chapter, where the same payoffs were used but were hypothetical. Nearly all subjects in the Allais problem chose the pair of options $a_2 - a_3$.

2.2.4 Violation of the reduction of compound lotteries axiom

A result emerging from the experiment is the shift in strategy choice and/or preference pattern of 30% of the subjects from the static to the pre-committed dynamic problems. A possible interpretation of this lies in considering the fact that this pattern of choice implies a violation of the reduction of compound lotteries axiom.

[2] Details are in Appendix V of Conlisk's paper.

A direct test of the violation of the reduction axiom in the context of the Allais paradox is present in Carlin (1992). Carlin tests violations of reduction in the Allais paradox (and common ratio effect problems) in a series of experiments, to see whether the evidence against the independence axiom is due to violations of independence only, or of independence and reduction together. He finds that

- violations of independence are reduced when one-stage problems are replaced by their probabilistically equivalent two-stage versions;
- violations of the reduction axiom are widespread, and are tested not to be random
- violations of reductions are of a kind which (weakly) reduces violations of independence.

A problem of violation of the reduction axiom in the experiment can occur from the fact that the prospects in the dynamic version of the Allais problem (Problems 3 and 4) are the probabilistic equivalent of the prospects in the static Allais decision problem (Problem 1). In problem 1 the agent is offered a choice of one prospect out of each of the two couples of prospects a_1 and a_2, and a_3 and a_4. In Problems 3 and 4 the agent has to pre-commit on how he wants to choose between the prospects a_1 and a_2 in Problem 3, and between the prospects a_3 and a_4 in Problem 4.

In Problem 3, a choice of option B, when combined with the initial probability, implies the prospect a_2 =(.20 chance of 10£; .75 chance of 5£; .05 chance of 0£). A choice of option A implies the prospect a_1=(1, 5£).

In Problem 4, a choice of B implies a_3=(.20 chance of 10£; .80 chance of 0£). A choice of A implies a_4=(.25 chance of 5£; .75 chance of 0£).

Choices consistent with EU are choices for a_1 and a_4 or a_2 and a_3. NEU choices are choices for a_1 and a_3 or a_2 and a_4.

Let us follow Carlin's reasoning in considering all the choice combinations which exhibit possible violations of the reduction axiom in the experiment.

Violation of reduction in the experiment occurs anytime the agent does not choose in the two-stage problems the prospects which are probabilistically equivalent to the prospects he has chosen in the one-stage problem. This would include the cases in which the agent's choice is

- consistent with EU in Problem 1 and with NEU in Problems 3 and 4;

- consistent with NEU in Problem 1 and with EU in Problems 3 and 4;

- consistent with either EU or NEU in all problems, but the preference pattern changes. That is, the agent's choice is consistent with either EU or NEU, but exhibits a different preference pattern in the two sets of problems.

In the following table the number of violations of reduction for the experiment is given.

Problem 1 - one stage		Problem 3 two stages		Problem 4		N° of reduction violations
NEU a_1 - a_3	A C	A	a_1	A	a_4	/
EU a_1 - a_4	A D	A	a_1	A	a_4	no violation
EU a_2 - a_3	B C	A	a_1	A	a_4	3
NEU a_2 - a_4	B D	A	a_1	A	a_4	/
NEU a_1 - a_3	A C	A	a_1	B	a_3	no violation
EU a_1 - a_4	A D	A	a_1	B	a_3	/
EU a_2 - a_3	B C	A	a_1	B	a_3	1
NEU a_2 - a_4	B D	A	a_1	B	a_3	/
NEU a_1 - a_3	A C	B	a_2	A	a_4	1
EU a_1 - a_4	A D	B	a_2	A	a_4	/
EU a_2 - a_3	B C	B	a_2	A	a_4	5
NEU a_2 - a_4	B D	B	a_2	A	a_4	no violation
NEU a_1 - a_3	A C	B	a_2	B	a_3	1
EU a_1 - a_4	A D	B	a_2	B	a_3	/
EU a_2 - a_3	B C	B	a_2	B	a_3	no violation
NEU a_2 - a_4	B D	B	a_2	B	a_3	2
				Total		13

Out of 40 subjects, 13 violate reduction in their choices in some way, while the choices of 27 subjects (26 EU and 1 NEU) do not exhibit any shift either in the preference pattern or in the strategy. (As seen above, 10 out of these 13 subjects adopted in the dynamic problems a strategy which they maintained all through).

As observed in the results, the number of violations of the independence axiom (the number of NEU choices) increases from the one-step reduced formulation to the two-step one, contrary to Carlin's and Conlisk's results (Conlisk uses a three-step version of the Allais example).

According to Carlin, rejection of reduction is stronger here, considering that "most subjects do not obey the reduction axiom in the context necessary to establish that the Allais-paradox violations are violations of the independence axiom alone" (page 223). Carlin considers those violations of reduction which reduce violations of independence, in the sense that they lead to a smaller number of the latter. He concludes that this kind of reduction violation is not random, and is systematically more likely to occur than other kinds.

In the context of our experiment, a reduction violation which reduces violations of independence is: a violation of independence in problem 1 which becomes a non-violation in Problems 3 and 4; a non-violation in Problem 1 which remains a non-violation in Problems 3 and 4.

Reduction violations which reduce violations of independence

Problem 1 - one stage		Problem 3 two stages		Problem 4		N° of reduction violations
EU a_1 - a_4	A D	B	a_2	B	a_3	/
NEU $a_1 – a_3$	A C	A	a_1	A	a_4	/
NEU $a_1 – a_3$	A C	B	a_2	B	a_3	1 (SC)
NEU $a_2 – a_4$	B D	A	a_1	A	a_4	/
NEU $a_2 – a_4$	B D	B	a_2	B	a_3	2 (SC)
EU a_2 - a_3	B C	A	a_1	A	a_4	3

Carlin's conclusions about reduction violations do not seem in the experiment here to be confirmed.

There are 6 over 13 violations of this kind. 3 cases of violations of reduction are changes in preference pattern of EU subject.

The other 3 violations of reduction correspond to choices consistent with Sophisticated Choice. However, violation of reduction in this case seems to be a consequence of adopting Sophisticated Choice. Consider this in more detail.

The SC agent who has chosen $a_2 – a_4$ in the static problem 1, prefers the lottery to the sure option in the direct choice between the two, and therefore forecasts that at the choice node of problem 3 his choice will be for the lottery. As this choice corresponds to pre-committing to a_3 in problem 3, $a_2 – a_3$ will be his choice in the two problems. This was the choice of two of the subjects. Thus, adoption of SC implies violation of reduction, as the subject shifts from a NEU choice in problem 1 to an EU choice in problems 3 and 4.

The SC agent behaves as NEU in the static problem 1, so that his choice would be for either $a_1 – a_3$ or $a_2 – a_4$. In problems 3 and 4, however, the agent will forecast his potential inconsistency at the decision node of one or the other of the two problems.

The SC agent who has chosen a_1 - a_3 in the static problem 1, prefers the sure option to the lottery in the direct choice between the two, and therefore forecasts that at the choice node of problem 4 his choice will go for the sure outcome. As this choice requires pre-committing to a_4 in problem 4, a_1 - a_4 will be his choice in the two problems. In the experiment, the subject undergoes a preference shift, as he chooses a_2 – a_3 instead. Thus, adoption of SC on the part of this subject implies a violation of reduction, through violation of strategy, from NEU to EU, and of preference pattern, from safe to risky options (the agent chose the lottery in problem 2).

From the cases here it seems that SC as a model of choice in the dynamic problems requires the agent with non-expected utility preferences to maintain dynamic consistency through violation of reduction. Moreover, SC behaviour itself in the context of this experiment can only be detected if this violation of reduction takes place. In the static problem 1, the SC agent behaves as NEU. In the dynamic problems 3 and 4, the agent chooses as an EU agent, as he forecasts his future inconsistency, and pre-commits to the consistent EU choice. The shift from the NEU to the EU choice from the static to the dynamic problems allows to detect the SC strategy.

Carlin's results led him to conclude that reduction violations which decrease independence violations are more likely to occur than any other kind of reduction violations. This does not seem to be confirmed in the experiment.

Consider now also those choice combinations that increase violations of independence, in the sense of leading to more of them, instead of reducing them.

Following the previous reasoning, reduction violations which increase violations of independence should be: a non-violation of independence in problem 1 which becomes a violation in Problems 3 and 4; a violation in Problem 1 which remains a violation in Problems 3 and 4.

Reduction violations which increase violations of independence

Problem 1 - one stage		Problem 3 two stages		Problem 4		N° of reduction violations
EU a_1 - a_4	A D	B	a_2	A	a_4	/
EU a_1 - a_4	A D	A	a_1	B	a_3	/
EU a_2 - a_3	B C	B	a_2	A	a_4	5
EU a_2 - a_3	B C	A	a_1	B	a_3	1
NEU a_1 - a_3	A C	B	a_2	A	a_4	1
NEU a_2 - a_4	B D	A	a_1	B	a_3	/

There are 7 over 13 violations of this kind, and 6 out of 7 imply a shift from a non-violation to a violation of EU. 1 implies the shift in preference pattern of a NEU subject.

This kind of violations of reduction implies a shift in choice from EU to NEU behaviour, and constitutes the more frequent violation of reduction in the experiment. The other violations are either preference shifts or violations of reduction consistent with the sophisticated choice strategy.

The results in the experiment do not allow to perform statistics on the randomness of the reduction violation, or to conclude that violations of reduction are of a kind that reduces violations of independence. They show that the shifts in strategy (all from EU to NEU) and/or preference pattern of the subjects from the static to the dynamic problems imply violation of the reduction axiom. In this context, violation of reduction helps to define sophisticated choice. Shifts in strategy from NEU in problem 1 to EU in problems 3 and 4 are all consistent with sophisticated choice.

There is another way in which changes in strategies might be interpreted. A problem of violation of the reduction axiom in the experiment could arise because the prospects in the dynamic version of the Allais problem are probabilistically equivalent to the prospects in the static Allais decision problem. However, the decision trees representing the dynamic decision problems 3 and 4 (the left and right handside trees in 1.2) are strategically equivalent to the tree representation of two static decision problems offering, respectively, a choice between a_1 and a_2, and between a_3 and a_4.. This is noted by Machina (1989) and refers to LaValle and Fishburn (1987). The shift in strategy and/or preference pattern could represent violation of the strategic equivalence between the static and the dynamic tree representation of the same prospects in the Allais problem.

2.3 Conclusion

3. problems imply violation of the reduction axiom. It emerges that violation of reduction helps also to define sophisticated choice: all shifts in strategy from NEU in problem 1 to EU in problems 3 and 4 are consistent with sophisticated choice. Another way in which changes in strategies might be interpreted lies in the strategic equivalence of the static and dynamic trees representing the Allais prospects.

Session	Problem 1	Problem 2	Problem 3	Problem 4
S1	B - C EU a_2-a_3	A EU a_1-a_4	A would not change	A would not change
	B - C EU a_2-a_3	B	B would not change	B does not change
	B - C EU a_2-a_3	B	B would not change	B does not change
	B - C EU a_2-a_3	B	B would not change	B does not change
S2	B - D NEU a_2-a_4	B	B	B would change to A
	B - C EU a_2-a_3	B	B	B
	B - C EU a_2-a_3	B MNEU a_2-a_4	B	A would not change
	B - C EU a_2-a_3	B	B would not change	B does not change
S3	A - C NEU a_1-a_3	A MNEU a_2-a_4	B (a_2) (inconsistent in P3?): does not change	A (a_4) does not change
	B - C EU a_2-a_3	B MNEU a_2-a_4	B (a_2) would not change	A (a_4) (inconsistent in P4?): would not change
	B - C EU a_2-a_3	B	B would not change	B does not change
	B - C EU a_2-a_3	B	B would not change	B does not change
S4	B - C EU a_2-a_3	B	B would not change	B does not change
	B - C EU a_2-a_3	B	B would not change	B does not change
	B - C EU a_2-a_3	B	B would not change	B does not change
	B - C EU a_2-a_3	B MNEU a_2-a_4	B (a_2) would not change	A (a_4) (inconsistent in P4?): does not change
S5	B - C EU a_2-a_3	B	B would not change	B would not change
	B - C EU a_2-a_3	B NEU a_2-a_4	B	A (a_4) (inconsistent in P4?): wants to change to B
	B - C EU a_2-a_3	A EU a_1-a_4	A	A (a_4) (inconsistent in P4?): wants to change to B
	B - C EU a_2-a_3	B	B would not change	B does not change

Session	Problem 1	Problem 2	Problem 3	Problem 4
S6	B - C EU a_2-a_3	B	B would not change	B does not change
	B - C EU a_2-a_3	B	B would not change	B does not change
	B - C EU a_2-a_3	B	B would not change	B does not change
	A - C NEU a_1-a_3	B SC a_2-a_3	B would not change	B does not change
S7	B - C EU a_2-a_3	B	B ----------	B ----------
	B - C EU a_2-a_3	B	B ----------	B ----------
	B - C EU a_2-a_3	B	B ----------	B ----------
	B - C EU a_2-a_3	B	B ----------	B ----------
S8	B - C EU a_2-a_3	B	B ----------	B ----------
	B - C EU a_2-a_3	B	B ----------	B ----------
	B - D NEU a_2-a_4	B SC a_2-a_3	B ----------	B ----------
	B - C EU a_2-a_3	B MNEU a_2-a_4	B ----------	A (a_4) (inconsistent in P4?): does not change
S9	B - C EU a_2-a_3	B	B would not change	B does not change
	B - C EU a_2-a_3	B	B would not change	B does not change
	B - C EU a_2-a_3	B	B (a_2) would change to A	B would not change
	B - C EU a_2-a_3	B	A would not change	A would not change
S10	B - C EU a_2-a_3	B MNEU a_1-a_3	A (a_1) (inconsistent in P3?): would not change to B	B (a_3) would not chan to A
	B - C EU a_2-a_3	B	B ----------	B ----------
	B - C EU a_2-a_3	B	B ----------	B ----------
	B - C EU a_2-a_3	B	B ----------	B ----------

2.2 Related experimental work

Two key references on the analysis of dynamic individual behaviour under risk will be considered, Cubitt, Starmer and Sugden (1998), and Paradiso and Hey (1999).

The work by Cubitt, Starmer and Sugden (1998) is concerned with

Paradiso and Hey (1999) are interested in a test of the agents' preferences for different temporal framings of the decision problem. CSS test whether the choice behaviour of the agents changes when the temporal framing of the decision problem changes. In particular, they test explicitly how appropriate is choice-time indifference as an assumption for a theory of individual dynamic choice.

In the experiment three choice problems have been played, which are strategically equivalent, that is, they imply the same opportunity set of probability distributions over the final outcomes, but differ with respect to the choice timing. The problems are represented by the following decision trees:

T_1

T_2

T_3

According to CSS, the more relevant result of their work is the violation of a condition which they term timing independence. they give two possible interpretations of this. violation of timing independence can be understood as sophisticated or mypic behaviour.

Then, it would be possible to characterise sophisticated choice as an approach which violates timing independence.

In the experiment considered in this chapter it is not possible to isolate timing independence as